THE

AËRONAUTICAL JOURNAL

(Founded 1897 in succession to the Annual Reports)

The Organ of the Aëronautical Society of Great Britain.

Telephone:
CITY 9761.

Telegrams:
"Didaskalos, Westrand, London."

Edited for the Council by

BERTRAM G. COOPER, A.F.Aë.S.

Published Quarterly.

Subscription per Annum, post free, 6s. 6d.

All Communications should be addressed to—
THE EDITOR, 11, Adam Street, Adelphi, W.C.

No. 74. APRIL—JUNE, 1915. Vol. XIX.

Notices of the Aëronautical Society of Great Britain.

Election.

April: S. L. Collins (Member).

Council, 1914-15.

The Council for the year 1914-15 is composed as follows:—Associate Fellows: A. E. Berriman, Griffith Brewer, Harris Booth, J. H. Ledeboer, Lieutenant A. R. Low, R.N.V.R., Squadron Commander F. K. McClean, R.N.A.S., Squadron Commander Alec Ogilvie, R.N.A.S., Mervyn O'Gorman, C.B., F. Handley Page, Dr. T. E. Stanton, Lieutenant-Colonel F. H. Sykes, Dr. A. P. Thurston. Members: Squadron Commander G. Aldwell, R.N., Colonel H. E. Rawson, C.B., R.E., Major-General R. M. Ruck, C.B., R.E., Dr. R. Mullineux Walmsley.

Library.

The Council desire to thank Mr. J. C. Hunsaker for a copy of his paper, "Scientific Aeronautic Research," and Mr. W. Ellis Williams for a copy of his paper, "On the Motion of a Sphere in a Viscous Fluid."

Edward Busk Memorial.

The Society has issued an appeal on behalf of the Edward Busk Memorial in the following form. Subscriptions, however small, will be welcome.

May, 1915.

Dear Sir (or Madam),

There is a strong feeling among members of the Staff of the Royal Aircraft Factory, the Aëronautical Staff of the National Physical Laboratory, members of the Aëronautical Society of Great Britain, and of King's College, Cambridge, and relatives and friends of Mr. Edward Teshmaker Busk, late of the London Electrical Engineers, that a memorial should be founded in his honour and for the promotion of the study of aeronautics, to the advancement of which science he so largely contributed.

Mr. Busk was only 28 years of age when, on the 5th November last, he died at Aldershot while flying his own stable aeroplane, owing to its destruction by fire, thus terminating a career already marked by fine achievement and full of promise for the future. At Cambridge he took First Class Honours in the Mechanical Sciences Tripos, and was awarded the John Wimbourne Prize and a Scholarship at King's College. After passing some years as an engineer with Messrs. Halls & Co., at Dartford, he joined the staff of the Royal Aircraft Factory, where he devoted his time especially to the mathematics and dynamics of stable flight of the full size aeroplane, to researches into the nature and cause of wind gusts, and to the uses of aircraft in warfare for offensive and defensive purposes. Besides this work, he was entrusted with the general control of the chemical, metallurgical, and physical research and test work at the Factory.

The Council of the Aëronautical Society have unanimously decided to award to Mr. Busk their gold medal, in recognition of his distinguished services to Aëronautical Science.

The Memorial will consist of (i.) a Studentship to enable a student to carry on some research in aeronautics or a kindred subject, and (ii.) a Lecture on some such subject to be given annually by the holder of the Studentship or by some other Lecturer, and to be published in the Aëronautical Journal.

Subscriptions to the amount of about £2,500 have been received or promised, and further contributions will be gratefully acknowledged by Sir Edward H. Busk, 11, Sussex Place, Regent's Park, N.W., or the Secretary of the Aëronautical Society of Great Britain, 11, Adam Street, Adelphi, W.C. Donors will kindly state on the enclosed form to which of the above purposes they wish their contributions to be applied.

Yours faithfully,

R. M. RUCK, Major-General,
Chairman of Council, Aëronautical Society of Great Britain.

MERVYN O'GORMAN,
Superintendent, Royal Aircraft Factory.

R. T. GLAZEBROOK,
Director, National Physical Laboratory.

M. R. JAMES,
Provost of King's College, Cambridge, and Vice-Chancellor of the University of Cambridge.

W. H. MACAULAY,
Fellow and late Tutor of King's College, Cambridge, and formerly University Lecturer in Applied Mechanics.

B. HOPKINSON,
Fellow of King's College, Cambridge, and Professor of Mechanism and Applied Mechanics.

ANNUAL GENERAL MEETING.

The ANNUAL GENERAL MEETING of the Aëronautical Society of Great Britain was held on Thursday, 20th May, 1915, at 8.15 p.m., at the Royal Society of Arts, John Street, Adelphi, W.C. The Chairman of Council, Major-General R. M. Ruck, C.B., R.E., presided.

The Secretary read the notice convening the meeting.

The Chairman asked that the meeting approve the postponement until that evening, owing to the war, of the date of the Annual General Meeting, which should, under Rule 11, have been held before the 31st March. The postponement was approved.

The Secretary then read the Council's Report for 1914-15.

The Society and the War.—In the four months of normal activity that followed last year's Annual General Meeting the Society gave evidence of maintaining and possibly exceeding the rate of progress of the immediately preceding years. On the advent of the war in August last, however, it was evident that the Society would be considerably affected. The inadequacy of its financial resources as compared with the activities required of it has been well known, and the very small balance left available for emergencies rendered it essential that all but the vital activities of the Society should be at once curtailed. It was consequently decided to hold no meetings except the Wilbur Wright Memorial Lecture and the Annual General Meeting. Fortunately the War Office recognised the value of the work done by the Society and generously approved of an arrangement whereby the services of the staff were utilised to a certain extent by the authorities, from whom a liberal grant is being received.

On the other hand the striking vindication of the national importance of aeronautics which the war has afforded cannot fail to be a source of much gratification to the members of the Society which owed its early existence mainly to the high aspirations of its members and their confidence in the coming triumphs of the science to which they were devoted.

The Society can congratulate itself on having done all in its power to convince both the public and the Services of the extreme importance of the subject, and the record of views and forecasts to be found in the reports of its meetings is of the greatest interest at the present time.

Members on Active Service.—More than one hundred members of the Society are on active service at the present time. Not a few have received special recognition for their services, amongst whom may perhaps be specially mentioned Major-General Sir David Henderson, K.C.B., D.S.O., Director General of Military Aeronautics. Nine members have died in the performance of services directly or indirectly connected with the war, viz.:—

Reginald Alston.
Lieutenant-Commander T. F. Besant, R.N.
2nd Lieutenant E. T. Busk.
Lieutenant S. Pepys Cockerell, R.F.C.
Lieutenant R. T. Gates, R.N.A.S.
Lieutenant C. S. Head, R.N.
The Hon. H. Lyttelton Pelham.
Major G. H. Raleigh, R.F.C.
Major H. Tyler, R.E.

Meetings.—Four Ordinary Meetings and the Wilbur Wright Memorial Lecture have been held since the date of the last report. The Society is much indebted to the Right Hon. Lord Sydenham, G.C.M.G., F.R.S., Major-General Sir David Henderson, K.C.B., D.S.O., and Mr. Horace Darwin, M.A., F.R.S., F.Ae.S., for their kindness in presiding at meetings. The fortnightly papers read before the Society had come to play so important a rôle in scientific aeronautical life that it is evident that it is very desirable that this branch of the Society's activities should be resumed at the earliest possible opportunity.

Wilbur Wright Memorial Lecture, 1914.—The second Wilbur Wright Memorial Lecture was read on 20th May, 1914, by Dr. R. T. Glazebrook, F.R.S., F.Ae.S., and attracted widespread attention to the important work on aeroplane design, especially as regards stability, that had been accomplished in this country.

The Banquet which was held on the night preceding the lecture was noteworthy for the number of distinguished guests present.

The Wilbur Wright Memorial Fund will, in a few years time, be enriched by the balance then remaining of the Aero-Educational Premiums Fund, and appears likely to reach the sum of £1,200, originally aimed at.

The third Wilbur Wright Memorial Lecture will be delivered by Prof. G. H. Bryan, F.R.S., on 20th May, 1915.

Technical Terms Committee.—The Technical Terms Committee mentioned in the last report, upon which sat official representatives of the Admiralty and the War Office, gave much consideration to a list of terms submitted by Mr. Mervyn O'Gorman, and eventually 98 definitions were drawn up and issued. Subsequently, on the initiative of the Comptroller-General of the Patent Office, an official representative of that Department was appointed to the Committee, which will resume its work at the first opportunity.

Prizes to Northampton Polytechnic Institute.—The sum of £5 was voted to the Governing Body of the Northampton Polytechnic Institute, Clerkenwell, for prizes to students at their Aëronautical Classes, and was allocated as follows :—

> Two class prizes of £2 and £1 respectively, and in addition a prize of the value of £2 for an essay upon a subject selected by the competitors from a list of subjects, of importance to aeronautics, approved by the Principal.

Associate Fellowship Election.—The fourth election of Associate Fellows, which had been deferred until the autumn of 1914, was, owing to the war, somewhat interfered with. Fifteen names were, however, added to the roll, which now numbers 73.

Library.—The re-organisation of the Library, on the approved lines mentioned in the last report, has occupied much time during the past year, and is well advanced. A card-index of books under their titles has been compiled, and the classification of the volumes is in hand.

One hundred and nine books and pamphlets were acquired during the year, and the number of lantern slides was also increased.

The Society's Gold Medal.—Two gold medals were awarded during the year, to Professor G. H. Bryan, F.R.S., and Mr. Edward T. Busk, B.A., respectively, and will be officially presented at the Annual General Meeting on 20th May, 1915. Both the awards were for important work on aeroplane stability, the work of the Society's members in connection with which has placed this country in the front rank of aeronautical development.

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The Lanchester Trophy for Model Gliders.—An important competition, having for its object the improvement of aerofoil efficiency, is being organised, for a trophy presented by Mr. F. W. Lanchester, and it is hoped to publish full details before long.

Tests of Models in Wind Channel.—The model competition to be undertaken, in co-operation with the Kite and Model Aeroplane Association, under the control of the Society's Research Committee, has been postponed for a time, but will be taken up again at an early opportunity.

Joint Committee with the Royal Meteorological Society.—Following on the action mentioned in the last report, a Joint Committee of the Aëronautical Society and the Royal Meteorological Society was formed, consisting of the following gentlemen :—Dr. T. E. Stanton, F.R.S., Lieutenant-Colonel F. H. Sykes, Alec Ogilvie and another (representing this Society), C. J. P. Cave, M.A., Dr. C. Chree, J. S. Dines, and G. K. Lempfert (representing the Royal Meteorological Society), with Colonel H. E. Rawson, C.B., as the Chairman of the Committee. Unfortunately the intervention of the war has prevented any work being undertaken.

Meteorological Conference at Edinburgh.—The above Conference, in the arrangements for which, as stated in the last report, the Society was co-operating, has, owing to the war, been adjourned *sine die*.

In Memoriam (Edward T. Busk).—Owing to the pre-occupation of Members of Council, due to the war, it has not yet been possible to prepare the Society's intended public appeal. It is hoped, however, that this appeal will be issued shortly, when full details will be given.

Finance.—It was stated in the last report that the financial position of the Society was much more satisfactory than for many years. Despite a falling off in subscription income, and in the number of new Members elected in 1914, owing to the war, there is every reason to anticipate that the Society will be able satisfactorily to weather the present crisis and to look forward to increased support in the future.

The balance of income over expenditure during the year was £57 12s. 6d., and the total cash credit on 31st December, 1914, was about £140.

The CHAIRMAN : You have heard the Report read, and a copy of the Balance Sheet has been circulated to every member. I now ask for any remarks on the Report and Balance Sheet.

There being none, I now propose that the Report and Accounts be adopted. Carried unanimously.

The CHAIRMAN : I now have to propose the following amendment to Rule 11, dealing with the election of the Council :—Add at end of Rule : " If no nominations of new candidates for the Council are received, a ballot paper shall not be circulated as provided above, and the retiring Members of Council shall be proposed for re-election at the Annual General Meeting." Carried unanimously.

The CHAIRMAN : I now propose, in accordance with the amendment to Rule 11, which we have just passed, that the retiring Members of Council, namely :—A. E. Berriman, Griffith Brewer, Squadron Commander Alec Ogilvie, R.N.A.S., Mervyn O'Gorman, C.B., F. Handley Page, Colonel H. E. Rawson, C.B., Dr. A. P. Thurston, Squadron Commander G. Aldwell, R.N., be re-elected for the current year. Carried unanimously.

The Annual General Meeting then terminated, and there followed:—

OFFICIAL PRESENTATION OF THE GOLD MEDAL OF THE AERONAUTICAL SOCIETY OF GREAT BRITAIN TO MRS. BUSK, ON BEHALF OF HER SON, THE LATE MR. E. T. BUSK, B.A., AND TO PROFESSOR G. H. BRYAN, F.R.S.

Major-General R. M. Ruck, C.B.: As Chairman of Council it becomes my duty this evening to make two presentations of the Society's Gold Medal, the highest honour which the Society can confer, and one which has been but rarely awarded. That great and original Pioneer of Flight, Wilbur Wright, whose memory we are met here to-night to commemorate, was one of the holders of this medal.

I am quite aware of my own deficiencies in representing this Society, but there may be something appropriate to these extraordinary times in what I may perhaps be allowed to call the figure-head being dressed in uniform. (Applause.) But there is something much more appropriate to the times in the fact that the two gentlemen whom we propose to honour (in one case it is only a memory) have by their genius and skill contributed much to the successful efforts which have been made by our gallant Flying Corps in France and elsewhere. (Loud applause.)

In order to appreciate properly the awards of the Society's Gold Medal to Professor Bryan and to the late Mr. E. T. Busk, it is necessary to understand something of the history of the study of aeroplane stability.

The "father" of the aeroplane was Sir George Cayley, a Yorkshireman, who in 1809 published the results of his aerial experiments, which had led to the construction of a large glider, or aeroplane without a motor. The records show that he had a practical knowledge of the features which it is necessary to have in aeroplanes for stability and steering, although, as far as we know, he did not publish any mathematical investigations on stability. The features he used are those which are used to-day in modern aeroplanes. Thus, more than a hundred years ago, an effective aeroplane was evolved that only required a light weight motor to be complete. From 1809 until December, 1903, when the Wright Brothers made the first controlled aeroplane flight, a very great deal of thought was given all over the world to the development of the flying machine and many theoretical and practical methods of investigating aeroplane stability were evolved, none, however, being quite satisfactory. During this time a method of mathematically investigating the motion of any moving body was also being evolved, and has been a "standard" method for many years past, ever since Routh published his "Rigid Dynamics" in 1860. The great honour of being the first man to apply this standard method to aeroplanes belongs to Professor Bryan. (Loud applause.) It must be remembered that when this work was commenced there were no actual full-size aeroplanes in existence, the basis of the investigations being model experiments. In June, 1903, Professor Bryan and Mr. Ellis Williams (another member of this Society) showed before the Society certain photographs they had obtained of the paths in the air of small model gliders, and later in the year read a paper on "The Longitudinal Stability of Aeroplane Gliders," founded on these model experiments. This work was the first attempt to apply Routh's Rigid Dynamics to flying machines. For several reasons the continuance of the investigations was delayed, and it was not until 1911 that the book "Stability in Aviation," dealing both with longitudinal and lateral stability, was published. By this time, of course, much flying had been done, but the stability investigations that had been carried out, with a few notable exceptions, were mostly founded on "rule-of-thumb." One notable excepiton was the work of Mr. F. W. Lanchester, who will take the chair at to-night's lecture. Mr. Lanchester took

OFFICIAL PRESENTATION OF GOLD MEDAL OF THE AËRONAUTICAL SOCIETY OF GREAT BRITAIN TO MRS. BUSK, ON BEHALF OF HER LATE SON (MR. EDWARD T. BUSK), AND TO PROF. G. H. BRYAN, F.R.S., AT THE ROYAL SOCIETY OF ARTS, JOHN STREET, ADELPHI, LONDON, W.C., 20th MAY, 1915, AT 8.40 P.M.

PROF. G. H. BRYAN, F.R.S. MR. F. W. LANCHESTER, M.I.C.E.

COL. W. S. BRANCKER, *Deputy Director of Military Aeronautics.* MRS. BUSK. MRS. BRYAN. MAJ.-GEN. R. M. RUCK, C.B., *Chairman of Council, Aëronautical Society.* MR. BERTRAM G. COOPER, *Secretary, Aëronautical Society.*

up the study of aeroplane stability, founded on model experiments, about 1892, and by very original methods, that bear very little superficial resemblance to Professor Bryan's, has obtained results similar to those given by Bryan's methods.

In the practical use of mathematical investigations of aeroplane stability, it is essential for success that certain experimental data shall be obtained in order to be able to calculate the dimensions and shape of full-size machines that will give the desired degree of stability. Professor Bryan was unable, owing to other calls on his time, to supply these most important data, and it has been indeed fortunate that in this country institutions existed where the necessary experiments could be carried on. One of these institutions is the National Physical Laboratory, which, as you may know, is a testing and research laboratory controlled by a committee of the Royal Society and assisted by a Government grant. The Director of the Laboratory is Dr. Glazebrook, a Fellow of this Society. When, in 1909, the Government appointed the Advisory Committee for Aeronautics, on which, by the way, Mr. Lanchester sits, to advise them on the development of aeronautics in the national interest, that committee turned to the National Physical Laboratory for the research work required, and as a result, an aeronautics section was started in the engineering section at the Laboratory, under the control of Dr. T. E. Stanton, F.R.S., now one of our Members of Council. This aeronautics section, at the immediate head of which is Mr. Leonard Bairstow, an Associate Fellow of this Society, has accomplished much notable work, especially in the case of aeroplane stability, the investigation of which, by Professor Bryan's methods, was first taken up at the Laboratory in 1911. The investigations are still proceeding, but over a year ago, as a consequence of such investigations and those at the Royal Aircraft Factory, it was possible to design an aeroplane in which the exact balancing of its various parts had been so calculated as the result of model and full-size experimental work that it should be inherently stable, and to show in actual flight that it fulfilled the designer's aims. And this brings me to the full-size experimental work that contributed to the design of this machine. That work was carried out by the late Mr. E. T. Busk, and earned him the Gold Medal that it will be my great pleasure shortly to present to Mrs. Busk.

The career of Edward T. Busk has been briefly sketched in the memorial appeal circular that you have in your hands. He joined the staff of the Royal Aircraft Factory in June, 1912, and, before actually going to the Factory, spent a month at the National Physical Laboratory, becoming familiar there with the aeronautical research carried on. (As you are no doubt aware, the National Physical Laboratory and the Royal Aircraft Factory are linked together through the Advisory Committee for Aeronautics.)

When Busk went to the Factory he immediately took up the question of the stability of the aeroplanes then being produced there. He found that in order to be able to utilise properly in full-size aeroplane design the data obtained at the National Physical Laboratory from model experiments, it was essential that actual experiments in the air should be systematically carried out. Full-size experiments have always been a feature of the Royal Aircraft Factory work. Acting on the principle that if you want a thing done well you should do it yourself, he determined to learn to fly and carry out the experiments himself. In this way he became the very rare combination of a theoretical designer of high merit and a skilful and daring pilot. There is no doubt that had he lived he would have accomplished very much more than the remarkable achievements he actually had to his credit. It is not possible at the present time fully to discuss these, but as you are aware, his researches into aeroplane stability, carried out in collaboration with Mr. Bairstow, were so far advanced in the autumn of 1913 that a completely stable aeroplane was then designed by him and demonstrated with entire success in November of that year. This was a remarkable achieve-

ment, because it was the first time that an aeroplane had been successfully produced that was the outcome of model and full-size experiments linked together by complete mathematical investigations of its stability. (Applause.)

Mr. Busk was a well-known member of this Society and not infrequently contributed towards its debates. His cheery personality was greatly appreciated by all who had the honour of his friendship. I would say a further word—Edward Busk gave his great abilities and his great energy to the work he loved so well—he gave more—he gave his life. Few of us here to-night but have lost some near relative or dear friend in this mighty convulsion. How do we feel these losses? Not, I think, quite in the ordinary manner, but rather in full and certain confidence that their devotion will not be wasted and that their memories will endure. So shall it be with Edward Busk.

General Ruck then presented the Gold Medal of the Society to Mrs. Busk, on behalf of her son, and to Professor Bryan, F.R.S., amid loud applause.

The recipients having made suitable acknowledgment, General Ruck announced that a memorial to Mr. Busk had been originated, the particulars of which were shown on the pamphlets which had been distributed to all present, and which had received much encouraging support.

Generally speaking, the intention was to commemorate the memory of Edward Busk by the encouragement of others to follow his distinguished example. It was hoped that adequate support would be given to make these rewards the most coveted prizes which are open to the student of scientific aeronautics.

General Ruck then introduced Mr. F. W. Lanchester, who was to preside at the Third Wilbur Wright Memorial Lecture, to be given that evening by Professor Bryan.

The FIRST MEETING of the FIFTIETH SESSION of the Aëronautical Society of Great Britain was held at the Royal Society of Arts, John Street, Adelphi, W.C., on Thursday, 20th May, 1915, at 8.50 p.m. Mr. F. W. Lanchester, M.I.C.E., presided over a large and distinguished audience.

WILBUR WRIGHT MEMORIAL LECTURE, 1915.

BY PROF. G. H. BRYAN, SC.D., F.R.S.

Professor BRYAN, in his opening remarks, referred to the Girton graduate who, having given a lecture on the care of children, was asked by one of the audience whether she was married, and on replying in the negative was told that the questioner had thought so from the lecture. He wished to say at once that he felt himself very much in the position of the Girton graduate in connection with actual flying, and wished to emphasise the danger of expressing opinions on subjects with which one was not fully familiar. For example, if anyone had asked him six months before for his opinion on the difference in behaviour while in circling flight between wings which were straight but bent up relatively to each other in what was known as a dihedral angle, and wings which were not straight but curved, he would have said that the behaviour of the two types would have been practically the same. This opinion would, however, as was very clearly shown by the calculations in the present lecture, have been absolutely wrong.

We had all heard lately the insistent cry, "Wanted—more shells." Those who would read his book, "Stability in Aviation," and in fact everything that he had published on the subject, as far back as 1897, would find that throughout all his work the insistent cry was to be found, "Wanted—more algebra."

As soon as one took up the mathematical study of the motions of a rigid aeroplane, one came at once to the six equations of motion of a rigid body, and was launched at once upon long algebraical calculations. Now he wanted to say at once that algebra was a subject which he loathed and detested. He had been made to hate it intensely when young, by being forced to spend his time solving the dry and profitless exercises in Todhunter's Algebra. Anything more depressing and hateful to the young student than these exercises he could not conceive.

He hated algebra and no doubt Mr. Lanchester also hated it, for in the investigations which he had published into aeroplane stability he had tried to do without algebra, and he, Professor Bryan, would like to say that Mr. Lanchester had obtained remarkable results and had carried the subject as far as any man could hope to do without using algebra. Professor Bryan had seen only too clearly from the commencement that unfortunately algebra could not be done without, and therefore had to face the distasteful task as best he could. He could not, however, owing to lack of time, do much work without an assistant. With the help of Mr. Ellis Williams he had been able to get a good deal done, but not nearly so much as he would have liked.

The problem with which he would deal that night should complete Volume I. of a treatise on the Rigid Dynamics of Aeroplane Motions, and was contemplated at the time when "Stability in Aviation" was written. At that time he was for several reasons very tired of the work, and the book was finished off as quickly as possible, the untouched work being lumped together at the end in "Problems." When, subsequently, he had obtained the assistance of Mr. Robert Jones, he had

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put him on to investigate Bent Planes as a change, and no doubt that his work on this subject would eventually be very valuable.

He wanted to explain the title of the lecture. He felt that a mathematical treatise on aeroplane stability should show by its title what it was. He wished to make quite clear that he was a theoretical and not a practical investigator into the problem, although it should be emphasised that his work was experimental just as much as that in wind channels or in the open air.

He had formed the opinion and had expressed it in his book that for ordinary commercial use of aeroplanes an inherently stable machine was required, in order that long distances could be covered, during which the pilot could be relieved of any strain in controlling the machine. He was still of that opinion, but in the case of war, where everything was dangerous, controllability, even at the expense of stability, would often be desirable, as it was essential that one should be able, in fighting in the air, to twist and turn very rapidly.

Since Mr. Robert Jones had left him he had not been able to obtain another assistant, and the present lecture had been written without assistance in the last Christmas vacation. He was glad to be able to say, however, that Mr. Bairstow of the National Physical Laboratory had very kindly, with the help of his assistants, amongst whom was Mr. Robert Jones—Professor Bryan's old pupil—gone through the algebra very carefully.

He wanted to point out that although the algebraical work was dry, the results obtained, when interpreted, were very interesting. For example, a minus sign in one of his equations might show that a rudder which one might have thought should be placed in front ought to be behind or *vice versâ*, and on looking into the matter one saw why. This interpretation of the results of the mathematics was the sort of work he liked to do.

Professor Bryan then summarised the lecture, the full text of which is printed below.

THE RIGID DYNAMICS OF CIRCLING FLIGHT.

(Steady Motion in a Circle—Lateral Steering of Aeroplanes).

Being the Third Wilbur Wright Memorial Lecture.

INTRODUCTION.

This paper is intended as a sequel to "Stability in Aviation" and deals with Problem 2 in Chapter XI. (p. 178). The long list of problems there given will show that before the book was written or even commenced, investigations had been projected and contemplated more than sufficient to fill a second volume on the Rigid Dynamics of Aeroplane Motions, my book on stability constituting the first. It will be found that the necessity of mathematical investigation in connection with stability and other aeroplane problems was first mentioned by me in "Science Progress" for October, 1897, and it would have been a great advantage if the completion of most of this work had preceded the first aeroplane flights. With reasonable facilities and assistance, longitudinal stability could have been completed by about 1900, lateral by 1904; the unwritten second volume would have been in the printers' hands by January, 1910. It has, however, been impossible to obtain the necessary facilities for the continuous pursuit of work of this kind consistently with due performance of the duties required in a department of a University College. Even if a class is placed in the hands of an assistant lecturer

and a large percentage of students fail, the trouble thus caused may involve the loss of a whole year in carrying on such calculations, as it is very difficult to go on with the heavier work unless continuous time can be given to it without any anxiety regarding the smooth working of one's professional duties. Serious obstacles have also been placed in the way of carrying out this work in other quarters, in several cases for no reason whatever. The encouragement which this work has received from the Aëronautical Society induced me to make an effort to undertake the present investigation, and thus to clear up unknown mysteries and to solve riddles which had puzzled me for a long time.

The problem of lateral steering was first considered by Reissner in 1910 (Zeitschrift fur Flugtechnik und Motorluftschiffahrt, 1910, p. 103), and it was again considered as a particular case of neutral equilibrium in a thesis by Dr. Karl Gehlen on Stability which was published soon after my book.*

This research took the form of a dissertation for the D.E. degree in Dr. Reissner's department at the Aachen Technical College.

Our methods were to a large extent identical, but Reissner's pupil performed calculations for actual aeroplanes, while it has been quite impossible for me to attempt anything of the kind at the time my book was published or indeed subsequently. As Dr. Gehlen's thesis is thus mainly occupied with stability the references to circular motion are very brief.

A German edition of "Stability in Aviation" was prepared by Dr. H. G. Bader (Berlin: Julius Springer, 1914). A paper on automatic rudders has been published recently by Crocco, but it is of little importance and the portions relating to stability are incorrect for reasons explained on p. 176 of my book.

In the following paper the notation, nomenclature, results and formulæ of "Stability in Aviation" will be freely used and referred to, as it would be impossible to give an independent treatment of this subject without re-writing a large amount of what has already been written. It is thus unfortunately necessary in this instance to follow the practice of many writers of mathematical papers who commence by quoting an obscure formula that has been published in an obscure journal, with the result that busy people cannot be at the trouble of reading their papers.

It is unnecessary to repeat much of what has been said in my "Introduction and Summary." As "deductions of conclusions from definite stated hypotheses," the present results are based on the following assumptions:—

1. That the systems considered consist of "narrow planes gliding at small angles" so that the air pressure on every surface-element obeys the sine law.

2. That if U_0 is the velocity in steady rectilinear flight, the additional velocity components in circular flight are small in comparison with U_0 so that the additional forces and couples can be expressed as linear functions of the "resistance derivatives" as in §§ 16, 17.

This assumption was perfectly legitimate when applied to the small oscillations discussed in "Stability." As applied to circular motion it will of course hold good, as an approximation, only in cases where the radius of the circular path is large compared with the linear dimensions of the machine. It also requires that the distance of the "turning point" from the centre of gravity should be small in comparison with the radius of the path. There are some systems in which this does not appear to be the case in the following investigation, but the systems in which these conditions occur appear to be undesirable.

* "Querstabilitat und Seitensteuerung von Flugmaschinen." Munchen: Druck von Oldenbourg. 1913.

I think it is probably possible to replace the present equations by more exact equations not involving the use of resistance derivatives and therefore better suited to the problem of steady motion in circles of small radius. In view of my limited opportunities for this work it would have been impossible to make any such attempt here. It is to be anticipated that the formulæ will prove to be more complicated.

3. It will be seen that in general the six equations of motion, longitudinal and lateral, are mutually interdependent, whereas we have in general confined our attention to satisfying the conditions of lateral equilibrium. In such cases it is to be anticipated from general considerations that the neglected conditions only produce small changes in the components of motion under consideration, but here again is a subject for further investigation.

It must be remembered, however, that it is often better to obtain roughly approximate formulæ which are capable of simple interpretation than to aim at greater accuracy and deduce a result which is not readily intelligible.

On the whole I should expect to find a fairly close *qualitative* agreement between the present conclusions and the results of experiment or practice, but of course differences of a *quantitative* character in the actual numerical working details must be expected.

One of the main objects of this investigation is to ascertain the conditions which render it easiest to steer an aeroplane in a horizontal circle of any radius that is not too small, and I have introduced the idea of " inherent controllability " to denote the property which a system may possess of freely describing a circular path without any pressure on the controlling rudders. In such cases the rudders will act as guides by preventing the aeroplane from leaving the chosen path, and as the system without them must, from the nature of the case, be wanting in asymmetric or lateral stability it is necessary for the working of such a system that the addition of the rudders should render it stable. The conditions for this must be worked out by the methods described in my book.

Inherent controllability is thus practically equivalent to asymmetrically neutral equilibrium as applied to the system minus the rudder, and it is determined by the condition that one root of the period equation for small lateral oscillations must vanish. It would of course be possible to avoid introducing a new term for this particular condition, but I think the distinction is convenient as the importance of the condition must necessarily be mainly in applications to the problem of control.

It will be found that there are several different ways of obtaining inherent controllability and that in circling flight the system turns about a point which in some cases is in front and in some cases behind the centre of gravity. The axis of x or horizontal line through the centre of gravity in the direction of forward motion thus envelopes a circle of radius, a, the " turning point " being the point of contact of the axis tangent with its envelope, and the lateral or sideways velocity of the aeroplane being proportional to the distance of the centre of gravity from the turning point. This length and the inclination of the aeroplane to the vertical constitute two independent variables which can be so chosen as to satisfy two conditions of lateral equilibrium, but as there are three, a third variable is in general required, and if a rudder plane is used, this latter variable may be taken to be the pressure on that plane. The condition for inherent controllability is that the three equations of lateral equilibrium should satisfy some further identical relation by which the number of variables is reduced to two, and there are several ways in which this may be done.

The results which are summarised at the end lead to some interesting

conclusions which were quite unexpected when I commenced writing the paper. In particular, they show the differences in behaviour between wings that are bent up and down respectively, the advantages, under certain circumstances, of curved wings as contrasted with plane wings bent into a simple dihedral angle, and generally that the form and curvature of the wing areas may play a much more important part in circling flight than I had anticipated. Indeed, it is fortunate that I have never expressed any opinion in discussions on this subject as it is now fairly certain that such an opinion would have been wrong.

The applications to the flight of birds are obvious and suggest much interesting material for discussion. At any rate a good many peculiarities in the wing structure of the circling birds appear to admit of interpretation on dynamical principles.

With regard to the possible application of these results to actual aeroplanes, it remains to be seen how far it is desirable or practicable to realise the conditions of inherent controllability in a real flying machine. But I cannot help thinking that a study of the present work, followed by a few experiments, will either lead to improvements in the steering of aeroplanes, or if the present arrangements are the best, it will now be easier to understand the reason why.

I have said very little about the use of *ailerons* in this paper, as I think it more desirable to ascertain what can be done without them. Their effects are after all much more easily studied than some of the problems here considered, and there is therefore less reason for my taking this matter up. In any case control by means of *ailerons* does not come within the province of problems on inherent controllability.

CHAPTER I.

Fundamental Principles.

I.—GENERAL EQUATIONS.

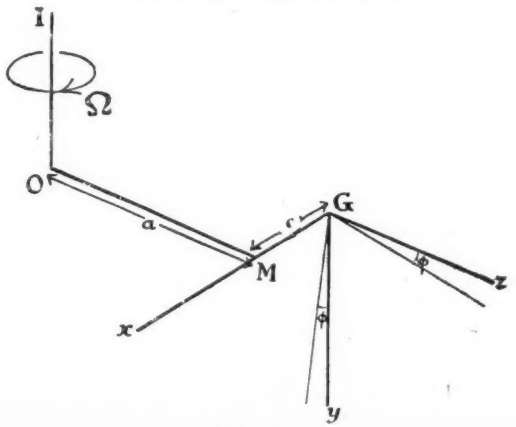


FIG. 1.

1. Let the plane of symmetry of the aeroplane meet the horizontal plane through its centre of gravity G in the line Gx. Let OI be the vertical axis about which the aeroplane is turning, O being the centre of the circle which it is

describing. Let OM, the perpendicular distance of O from Gx be equal to a, and let $GM = c$ so that the aeroplane is turning about a point in front of its centre of gravity by an amount c. Then if we take Gx and parallels through G to IO, OM as axes and suppose Ω to be the angular velocity about OI, the component velocities of G will be

$$\Omega a, \quad O, \quad \Omega c$$

while the component angular velocities will be

$$O \quad \Omega \quad O,$$

The aeroplane will however heel over through some angle ϕ, and we must therefore refer its motion to axes of y and z making angles ϕ with the vertical and horizontal respectively. The components of linear and angular velocity now become

$$\left.\begin{array}{llll} u = a\Omega, & v = c\Omega \sin\phi, & w = c\Omega\cos\phi \\ p = O, & q = \Omega\cos\phi, & r = -\Omega\sin\phi \end{array}\right\} \quad . \quad . \quad . \quad (1)$$

Substituting in the equations of motion of page 23, §15, we obtain (with $\theta = o$)

Longitudinal Group.

$$W\frac{c\Omega^2}{g} = H - X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2u)$$

$$-W\frac{a\Omega^2\sin\phi}{g} = W\cos\phi - Y \quad . \quad . \quad . \quad . \quad (2v)$$

$$F\frac{\Omega^2\cos^2\phi}{g} = -Hh - N \quad . \quad . \quad . \quad . \quad . \quad (2r)$$

Lateral Group.

$$-Wa\frac{\Omega^2\cos\phi}{g} = -Z - W\sin\phi \quad . \quad . \quad . \quad . \quad (2w)$$

$$(B-C)\frac{\Omega^2\sin\phi\cos\phi}{g} = -L \quad . \quad . \quad . \quad . \quad . \quad (2p)$$

$$F\frac{\Omega^2\sin\phi\cos\phi}{g} = -M \quad . \quad . \quad . \quad . \quad . \quad . \quad (2q)$$

In these equations $a\Omega$ represents the velocity of the aeroplane in the forward direction, but the resultant velocity is $\Omega\sqrt{(a^2+c^2)}$ and the radius of the circular path is $\sqrt{(a^2+c^2)}$.

Equations $(2u, v, r)$ show that the conditions of longitudinal equilibrium are affected by the circling motion and are not the same as in rectilinear motion. The differences can evidently be compensated for by tilting the aeroplane about the axis of z, by changes in the velocity of propulsion and by manipulating the longitudinal rudder planes. The effect of tilting will be that the axis of x in the above equations will occupy a different position in the aeroplane to the direction of steady rectilinear motion. We may of course use equations (2) to discuss the conditions of lateral equilibrium on this understanding, but the moments of inertia and resistance derivatives will then have to be referred to the normal axes by means of the formulæ of transformation of Rigid Dynamics and three dimensional formulæ corresponding to those of p. 48, § 31.

2. The alternative is to turn the axes round with the aeroplane through an angle θ, which we take to be positive if the aeroplane is depressed in front; this convention being necessary in order that the translational and rotational displacements should be referred to a right-handed system of axes.

In this case we have for the components of

Linear Velocity.	Angular Velocity.	Gravity.	
$u = \Omega\,(a\cos\theta + c\sin\phi\sin\theta)$	$p = \Omega\cos\phi\sin\theta$	$W\cos\phi\sin\theta$	
$v = \Omega\,(-a\sin\theta + c\sin\phi\cos\theta)$	$q = \Omega\cos\phi\cos\theta$	$W\cos\phi\cos\theta$	(3)
$w = +\Omega c\cos\phi$	$r = -\Omega\sin\phi$	$-W\sin\phi$	

The equations of equilibrium now assume the forms

Longitudinal Group.

$$W\frac{\Omega^2}{g}(c\cos\theta - a\sin\phi\sin\theta) = H - X + W\cos\phi\sin\theta \quad . \quad . \quad \dagger(4u)$$

$$W\frac{\Omega^2}{g}(-c\sin\theta - a\sin\phi\cos\theta) = -Y + W\cos\phi\cos\theta \quad . \quad . \quad (4v)$$

$$\tfrac{1}{4}(B - A)\sin\theta\cos\theta + F(\cos^2\theta - \sin^2\theta)\frac{\Omega^2}{g}\cos^2\phi = -Hh - N \quad . \quad (4r)$$

Lateral Group.

$$-W\frac{\Omega^2}{g}a\cos\phi = -Z - W\sin\phi \quad . \quad . \quad . \quad \dagger(4w)$$

$$\left\{(B - C)\cos\theta - F\sin\theta\right\}\frac{\Omega^2}{g}\sin\phi\cos\phi = -L \quad . \quad . \quad . \quad (4p)$$

$$\left\{(C - A)\sin\theta + F\cos\theta\right\}\frac{\Omega^2}{g}\sin\phi\cos\phi = -M \quad . \quad . \quad . \quad (4q)$$

These equations are, however, much less convenient than equations (2) and therefore we shall not use them.†

3. An important object to be kept in view is the desirability of reducing as far as possible the stresses on any rudder planes that are remote from the main lifting planes of the machine. *Ailerons*, on the other hand, are of course attached to the strong framework carrying the main planes, and they may act not only directly, but to some extent indirectly by modifying the pressure on the parts of the main planes in their immediate neighbourhood. But in every case *inherent controllability*, as we may call it, is secured by making the resistances to circular motion small, and it may be desirable to see whether it is possible to make them vanish. The action of the rudder planes will then determine the character of the path. If these planes were removed the equilibrium of such an aeroplane would be asymmetrically or laterally neutral, but the rudder planes would maintain lateral stability and when these rudders were turned round the aeroplanes would describe a circular path in which the rudders would move tangentially, thus experiencing no resistance. It remains to be seen how far it is possible to approximate to these conditions.

4. In the following investigations we shall resort to approximate methods by assuming that the radius of the circular path is large compared with the dimensions of the machine, and that the forces and couples due to air resistance differ from those in steady motion by small quantities that may be represented in terms of the system of resistance derivatives. We employ the equations (2), in which the axis of x is horizontal and we will also write

$$a\Omega = U = U_0 + u$$

where U_0 is the corresponding velocity in a standard state of steady rectilinear

† The components of W assume a different form in these equations from those on p. 23, because we have rotated the axes first through Φ and then through θ instead of in the reverse order. Our Φ and θ are thus different from those of § 15, though the difference is negligible if they are small.

motion in the direction of the axis of x supposed to remain horizontal. We use the symbols X_1 Y_1 Z_1 L_1 M_1 N_1 to denote the forces and couples which have to be impressed on the system by means of the rudder planes in order to maintain equilibrium in the circular path, these being the quantities which it is desired to make small or vanish. The component X_1 would not be easy to produce by means of rudders, but it can easily be produced by a slight variation in the propeller thrust. Taking account of the conditions of equilibrium for rectilinear motion, viz., $H - X_0 = 0$, $W - Y_0 = 0$, $Hh - N_0 = 0$ we get

Longitudinal Group.

$$Wc\Omega^2/g = -uX_u - \Omega \sin\phi\,(cX_v - X_r) - X_1 \quad (5u)$$
$$-Wa \sin\phi\Omega^2/g + W(1 - \cos\phi) = -uY_u - \Omega \sin\phi\,(cY_v - Y_r) - Y_1 \quad (5v)$$
$$F\Omega^2 \cos^2\phi/g = -uN_u - \Omega \sin\phi\,(cN_v - N_r) - N_1 \quad (5r)$$

Lateral Group.

$$-W \cos\phi a\Omega^2/g + W \sin\phi = -\Omega \cos\phi\,(bZ_w + Z_q) - Z_1 \quad (5w)$$
$$(B - C) \sin\phi \cos\phi\Omega^2/g = -\Omega \cos\phi\,(bL_w + L_q) - L_1 \quad (5p)$$
$$F \sin\phi \cos\phi\Omega^2/g = -\Omega \cos\phi\,(bM_w + M_q) - M_1 \quad (5q)$$

If the propeller thrust passes through the centre of gravity N_u vanishes in (5r).

Where auxiliary surfaces are used for steering they are not to be included in the expressions for the resistance-derivatives, but their whole effects are to be included in the terms $X_1 \ldots M_1$.

CHAPTER II.

Straight Planes and Vertical Rudders.

II.—STRAIGHT PLANES.

5. Here by § 77 the only resistance-derivatives which do not vanish are

$$X_u = 2KSU \sin^2\alpha \qquad X_v = KSU \sin\alpha \cos\alpha$$
$$Y_u = 2KSU \sin\alpha \cos\alpha \qquad Y_v = KSU \cos^2\alpha$$
$$[L_p = KUI \cos^2\alpha] \qquad L_q = -2KUI \sin\alpha \cos\alpha$$
$$[M_p = -KUI \sin\alpha \cos\alpha] \qquad M_q = 2KUI \sin^2\alpha$$

or in the "$S^1\mu$" notation

$$X_u = 2KS^1U\mu^2 \qquad X_v = KSU\mu$$
$$Y_u = 2KS^1U\mu \qquad Y_v = KS^1U$$
$$[L_p = KUI^1] \qquad L_q = -2KUI^1\mu$$
$$[M_p = -KUI^1\mu] \qquad M_q = 2KUI^1\mu^2$$

of which L_p, M_p do not occur in our equations and are therefore enclosed in square brackets.

Commencing with equations (5r) and (5q) we see that these give with $U = a\Omega$,

$$N_1 = -F\Omega^2 \cos^2\phi/g \quad . \quad . \quad . \quad . \quad (6r)$$
$$M_1 = -F\Omega^2 \sin\phi \cos\phi/g - 2KIa\Omega^2 \sin^2\alpha \cos\phi \quad . \quad . \quad (6q)$$

and as ϕ is in general a small angle, it is obviously best to make $F = 0$ and $N_1 = 0$ rather than to attempt to make $M_1 = 0$.

If k is the radius of gyration of the surface S, so that $I = Sk^2$ we now get from the conditions of equilibrium

$$M_1 = -2W \tan\alpha \cos\phi/k^2/a \quad . \quad . \quad . \quad (7q)$$

for the couple about the axis of z due to the action of the rudders.

If the conditions $Z_1 = 0$, $L_1 = 0$ are to be satisfied we must have

$$\tan \phi = a\Omega^2/g = U^2/ga \quad . \quad . \quad . \quad . \quad (6w)$$

$$(B - C) \sin \phi = 2aKI^1\mu g \quad . \quad . \quad . \quad . \quad (6p)$$

Equation (6w) represents the fact that the component of gravity along the axis of z balances the centrifugal force just as in the conical pendulum. (6p) represents the condition that what is sometimes called the "centrifugal couple" due to rotation about a line which is not parallel to a principal axis balances the turning moment due to the excess of pressure on the outer wing which is moving the more rapidly, and in giving prominence to this equation the object is to ascertain how far it is possible to dispense with *ailerons*.

Now the equation (6p) requires that sin ϕ should be proportional to a, other things being the same, that is that

$$a \operatorname{cosec} \phi = \frac{B - C}{2KI^1\mu g} = \text{constant} \quad . \quad . \quad . \quad . \quad (7p)$$

The interpretation of this is that if the plane of symmetry of the aeroplane meets the vertical OI in a point Z, then MZ is always equal to the constant of (7p). Thus keeping Z fixed, the various possible circular paths will be such that the path which M always describes is a circular section of a fixed sphere through the centre of which the plane of symmetry passes. If G coincided with M as in the Case I. we might imagine the aeroplane tied by a rope or, better, attached by a rigid connection to the point Z, and it may be convenient to think of boys swinging round a maypole in order to picture the possible motions.

On the other hand, the conditions of equilibrium, in rectilinear motion, give

$$W = KS^1U_0^2\mu$$

assuming that the system is a single lifting one with neutral tail. From these we get

$$a \operatorname{cosec} \phi = \frac{r_2^2 - r_3^2}{2k^2} \frac{U_0^2}{g}$$

$$= \frac{r_2^2 - r_3^2}{k^2} \times \text{velocity height corresponding to } U_0 \quad . \quad (8p)$$

where k is the radius of gyration of the surfaces, r_1, r_2, r_3 the radii of gyration of the masses of the aeroplane.

If now, we try to combine this equation with the condition $Z_1 = 0$ we get

$$\frac{\sin^2\phi}{\cos\phi} = \frac{U^2}{U_0^2} \cdot \frac{2k^2}{r_2^2 - r_3^2}$$

$$\doteq \frac{2k^2}{r_2^2 - r_3^2} \text{ if } U \doteq U_0 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and the angle ϕ will therefore not be small. The radius a will therefore not be large compared with the velocity height U^2/g.

6. The only way, therefore, of making the machine describe curves of different radii will be by varying c so as to increase the radius from a to $\sqrt{(a^2 + c^2)}$.

Although in small oscillations there are no resistance-derivatives X_w, Y_w, N_w, it is clear that the effects of the lateral velocity w or Ωc only vanish to the first order in w, and that when w is large it may have a very considerable effect on the longitudinal motion, the extra terms thus introduced into the equations being of the order w^2 instead of w. In fact, according to the sine law of resistance, we should take the resultant thrust on the plane to be

$$KS\Omega\sqrt{(a^2 + c^2)} \times (a\Omega \sin \alpha + c\Omega \sin \phi \cos \alpha)$$

and similarly the value of U should be taken as $\Omega\sqrt{(a^2 + c^2)}$ instead of

Ωa, in forming the expressions for the lateral couples L, M, thus increasing them in the ratio of $\sqrt{(a^2+c^2)}$ to a.

With regard to the longitudinal equations $(5u)$ and $(5v)$ these need not occasion much difficulty. A small change uY_u in the velocity U will be sufficient to make Y_1 vanish, and the necessary value of X_1 in $(5u)$ can be obtained by a small variation in the propeller thrust; X_1 representing the (positive or negative) decrease in the thrust. By substituting in $(5v)$ from $(6w)$ and the conditions of equilibrium, I obtain

$$\frac{2u}{U} = \sec\phi - 1 - \frac{c}{\sqrt{(a^2+c^2)}}\sin\phi\cot\alpha \quad . \quad . \quad . \quad (10)$$

In $(5u)$ the term $Wc\Omega^2/g$ represents the component of centrifugal force along the axis of x, which of course has to be balanced by the thrust of the engine.

III.—EFFECTS OF THE RUDDER.

7. So long as the only vertical auxiliary planes are placed on the same level as the centre of gravity and not above it, these will have no effect on equations $(6p)$, $(7p)$, $(8p)$, and the only effects will be to modify the condition of equilibrium resulting from $(5w)$ which will no longer assume the form $(6w)$.

Now we have found that a rudder plane must exert a couple M_1 given by $(7q)$ in order to turn the aeroplane, and if this rudder is placed at a distance l behind the centre of gravity, M_1/l will represent the thrust Z_1 along the negative direction of the axis of Z.

Substituting in $(5w)$ we get

$$-\frac{a\Omega^2}{g} + \tan\phi = +2\tan\alpha\,\frac{k^2}{al}$$

$$a\tan\phi = \frac{U^2}{g} + 2\frac{k^2}{l}\tan\alpha \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The condition of equilibrium along the axis of Z is therefore the same as it would be if the rudder were removed and the "velocity-height" $(U^2/2g)$ increased by $k^2\tan\alpha/l$. If the rudder is placed in front, the effect would be equivalent to a decrease in the "velocity-height." Equation (9) will therefore hold good with the modification that $U^2 + 2g\tan\alpha\, k^2/l$ must be substituted for U^2 in it. In order to reduce the value of ϕ given by (9) it will be necessary to make l negative and small. This will certainly improve the controllability and in any case it will be an advantage to have the rudder in front rather than behind. But decreasing l merely means that we are correspondingly increasing the thrust on the rudder, the moment of this thrust being in every case determined by $(7q)$.

IV.—STRAIGHT PLANES WITH BOXED-IN ENDS OR VERTICAL PARTITIONS.

8. This is an arrangement seen in many early aeroplanes (compare Fig. VII., p. 143). If these auxiliary surfaces are not raised, but situated in the axis of z, the effect is to produce a force along this axis represented by the term $-wZ_w$ only. If T is the total area of these surfaces, however, the exact expression according to the sine law is

$$-K^1T\Omega\sqrt{(a^2+c^2)}\times c\Omega\cos\phi \quad . \quad . \quad . \quad (12)$$

Substituting in $(5w)$ we now have

$$-Wa\Omega^2/g + W\tan\phi = -K^1T\Omega^2c\sqrt{(a^2+c^2)} \quad . \quad . \quad (13)$$

With $a\Omega = U$ and the help of the equation of equilibrium in rectilinear motion, this gives

$$\tan\phi = \frac{U^2}{ga} - \frac{U^2}{U_0^2}\frac{c\sqrt{(a^2+c^2)}}{a^2\sin\alpha\cos\alpha}\frac{K^1T}{KS} \quad . \quad . \quad . \quad (14a)$$

so that the value of $\sin^2\phi/\cos\phi$ given by $(6w)$ will now have to be multiplied by the additional factor

$$1 - \frac{g}{U_0^2}\frac{c\sqrt{(a^2+c^2)}}{a\sin\alpha\cos\alpha}\frac{K^1T}{KS}$$

$$= 1 - \frac{c\sqrt{(a^2+c^2)}}{2ha\sin\alpha\cos\alpha}\frac{K^1T}{KS} \quad . \quad . \quad . \quad . \quad . \quad (14b)$$

where h is the velocity-height corresponding to U_0. It is evident that by making c considerable and a small this equation can be reconciled with $(8p)$ without making ϕ too large. In this case the pressure on the fins has a considerable effect in maintaining the curved path. On the other hand, if we wish to make a large the angle ϕ will necessarily be large by $(8p)$ and equations $(14a)$ $(14b)$ will only be satisfied by making c negative, so that the aeroplane now turns about a point behind its centre of gravity. It would thus appear that while the inclination ϕ and radius a are connected by $(8p)$ there are two ways of balancing gravity and centrifugal force in describing a circle of radius $\sqrt{(a^2+c^2)}$; one being to make a and ϕ small, when the aeroplane will turn about a point in front of its centre of gravity and be kept in its circular path by inward pressure on the partitions; the other to make a and ϕ large, when the aeroplane will turn about a point *behind* its centre of gravity and will be kept from falling inwards by the outward pressure on the partitions. In Case I., which represents the limiting form of Case III., for which $T \longrightarrow 0$, c does not enter into the equations of equilibrium and its value is therefore only determined by the value of $\sqrt{(a^2+c^2)}$ the radius of the circle to be described. For any given radius greater than a there are therefore two values of c which are equal and opposite.

9. Now I have worked out the solution of the simultaneous equations $(8p)$ $(14a)$ on the assumption that $\sqrt{(a^2+c^2)}$ the radius of the circle is to be a given constant R and that all the other data such as U_1 U_0 are constant. The easiest plan is to put $a = R\cos\theta$, $c = R\sin\theta$ and then eliminate θ. The result is a biquadratic in $\cos\phi$, and it is easy to verify that when T is small and R not too small this biquadratic has at least two real roots which make c positive and negative respectively, corresponding to the two possible motions. But in the limiting case the equation for ϕ degenerates to (9), and if this is written in the form of a quadratic in $\cos\phi$, the product of the roots will be -1 so that one value of $\cos\phi$ will be negative and greater than unity. This shows that two of the roots of the biquadratic will in general represent impossible values.

Suppose in the first place that c is positive so that the aeroplane turns about a point in front of its centre of gravity. If then a rudder plane be fixed at a distance l behind the centre of gravity the pressure on the rudder will tend inwards and therefore tend to prevent the aeroplane from turning about the vertical axis unless the plane of the rudder is turned through an angle whose tangent is greater than

$$\frac{c+l}{a}\cos\phi$$

As, however, we have seen that the value of a is limited, so that large circles can only be described by making c sufficiently large, it is clear that difficulties will occur in steering in every case.

Next suppose that c is negative so that the aeroplane is turning about a point behind its centre of gravity. If this point happens to coincide with the position of the rudder plane the direction of motion of this point will always be

XIIM

tangential to the rudder plane. The circular motion will therefore not be possible if the rudder plane is turned, until the pressure on it is sufficient to satisfy equations (7q). If the turning point is behind the rudder, as it will be when c is made large, the pressure on the rudder will tend to turn the aeroplane round even in the neutral position, so that it may be possible that the rudder will have to be turned inwards in order to prevent the aeroplane from turning too much in that direction.

The best way out of the difficulty would be to have two rudder planes, one in front and one behind. Theoretical considerations seem, therefore, to point to the conclusion that any aeroplane without raised fins or bent-up wings or *ailerons* may be expected to rock from side to side with a swaying motion, and though it may be quite easy to turn them round for a short distance it would be very difficult to steer them steadily and continuously round a circle of large radius.

V.—RAISED FINS OR RUDDERS.

10. In the previous investigations the main difficulty arises from the fact that the condition $L_1 = 0$ requires that sin ϕ shall be proportional to a, in other words, that the couple $(B - C) \sin \phi \cos \phi \Omega^2 / g$ becomes small in comparison with $-\Omega \cos \phi L_q$ when ϕ is small and a large. The difficulty could of course be overcome by the use of raised auxiliary surfaces by means of which a suitable couple L_1 could be impressed on the system. Inasmuch however as $L_q = -M_q \cot a$ and that a is small it will be seen that the couple necessary to produce this effect would be large compared with the couple M_1 required to produce equilibrium of moments about the axis of y. If a single auxiliary plane were used for both purposes, its height would have to be large in comparison with its horizontal distance behind the centre of gravity. The difficulty is very similar to that described in connection with the effect of a single vertical fin on stability (§78). So far as it is a mere question of reconciling the two conditions (5p) and (5q) any difficulty could be got over by the use of two auxiliary planes instead of one. On the other hand, the comparative largeness of L_q makes any such attempt undesirable in view of the assumed objection to setting up large stresses in parts of the machine that are remote from the lifting surfaces. This objection is not as bad, however, as might be otherwise supposed, because when Ω is small the stresses are in no case large, whereas when the aeroplane is turning round a sharp curve, the term $(B - C) \sin \phi \cos \phi \Omega^2 / g$ becomes operative, and for very sharp curves it may even be necessary to apply a couple in the reverse order owing to this term becoming larger than is required to maintain equilibrium of moment about the axis of x.

In view, however, of the fact that stabilisers can be substituted for fins, and that these and bent-up wings can certainly produce at least the same variety of effects that are obtainable with fins, it is no longer necessary further to discuss the effects of auxiliary surfaces remote from the main planes.

CHAPTER III.

Bent-up and Curved Planes.

VI.—BENT-UP PLANES.

11. The formulæ for the resistance-derivatives of a pair of bent-up wings of § 75, p. 125, and § 95, p. 154, can be most easily adapted to the present purpose by assuming the origin used in calculating them to be the point in the axis of x about which the machine turns. The value of x at the centre of gravity will

thus be the distance of this turning point behind the centre of gravity and will be equal to minus c of the previous investigations. At the same time as the *equations of motion* are to be referred to the centre of gravity, it will be necessary to substitute the value of $M_q + xZ_q$ given by the formulæ for M_q. We may write $x = x_0 + x^1$ where x^1 is the distance of any point on the wing in front of the centre of gravity, and this precaution is necessary if we wish to take account of wings of various shapes such as are found in birds, though in ordinary cases x^1 will vanish and x and x_0 will be identical. Still, had this term been overlooked at the present stage, there would have been serious risk of mistakes being made in some future investigation. We now obtain

$$0 = \frac{N_0}{KU^2} = \int l\ (mx^1 - ly)\ dS = \int \mu\ (x^1 \cos\beta - \mu y)\ dS^1 \quad . \quad . \quad (16r)$$

$$\frac{Z_q}{KU} = \int n\ (2lz - nx)\ dS = \int \sin\beta\ (2\mu z - x \sin\beta)\ dS^1 \quad . \quad . \quad (16w)$$

$$\frac{L_q}{KU} = \int (ny - mz)\ (2lz - nx)\ dS = -\int (z\cos\beta - y\sin\beta)\ (2\mu z - x\sin\beta)\ dS^1 \quad . \quad . \quad . \quad (16p)$$

$$\frac{M_q}{KU} = \int (lz - nx^1)\ (2lz - nx)\ dS = \int (\mu z - x^1 \sin\beta)\ (2\mu z - x\sin\beta)\ dS^1 \quad (16q)$$

or with $x^1 = 0$

$$\frac{M_q}{KU} = \int lz\ (2lz - nx)\ dS = \int \mu z\ (2\mu z - x\sin\beta)\ dS^1 \quad . \quad . \quad (16qr)$$

where $l = \sin\alpha$; $m = \cos\alpha\cos\beta$; $n = \cos\alpha\sin\beta$; $dS^1 = dS\cos^2\alpha$; $\mu = \tan\alpha$. Now it will be seen at once that L_q can be made to vanish by taking

$$\int (z\cos\beta - y\sin\beta)\ (2\mu z - x\sin\beta)\ dS^1 = 0.$$

As in Mr. Harper's theory of "Antoinette" stability, we write

$$z\cos\beta - y\sin\beta = \zeta = \rho + \epsilon\ ;\ z\sec\beta = \rho$$

and obtain

$$x = \frac{2\int \rho(\rho+\epsilon)\tan\alpha\cos\beta dS^1}{\int(\rho+\epsilon)\sin\beta dS^1} \quad . \quad . \quad . \quad . \quad (17)$$

This expression is identical with Mr. Harper's limit of stability, p. 163, § 100, equation (183), and the interpretation is very simple. If the rudder is placed in front of this turning point the pressure of the air on it will act on the inner side and its moment will tend to increase the rotation. If the rudder is behind the turning point the moment of the air pressures on it will tend to retard rotation.

The advantage of the bent-up planes will be immediately obvious. Owing to the possibility of making L_q small the value of $\sin\phi/a$ required by the equation of moments about the axis of x need no longer be constant, and it is thus possible to turn in a circle of large radius without making ϕ large or making the axis of x tangentially envelop a circle of small radius a. A small change in the value of x will produce a corresponding change in L_q and enable us to satisfy the condition (5p), namely,

$$(B - C)\sin\phi\cos\phi\Omega^2/g = -\Omega\cos\phi L_q$$

It remains to consider the effect on the other conditions of equilibrium, and for this purpose we have to consider particular cases.

VII.—SIMPLE DIHEDRAL ANGLE BETWEEN RECTANGULAR PLANES.

12. By this we mean to say that the two wings are plane, that they form a dihedral angle ($180° - 2\beta$ approximately), but that they are not themselves curved, so that β as well as α is constant at every point of a wing, and that the shape of each wing is approximately rectangular. In this case factors involving β and α can be placed outside the signs of integration. It will be seen at once that the conditions for $L_q=0$ and $M_q=0$ become identical if $\epsilon=0$, that is if the dihedral angle is situated at the centre of gravity instead of being raised above it.

Having got rid of L_p and M_p, the equation of moments about Ox gives

$$(B - C) \sin \phi \cos \phi = 0 \quad . \quad . \quad . \quad . \quad (18)$$

which is satisfied by $\sin \phi = 0$ or $B = C$.

If $B = C$ there only remains the equation $(5w)$, which gives

$$\tan \phi = \frac{U^2}{a}\left(\frac{1}{g} - \frac{K}{W}\int \sin \beta \ (2\mu z - x \sin \beta) \ dS^1 \right)$$

$$= \frac{U^2}{ag} - \frac{U^2}{U_0^2 a} \frac{\int \sin \beta \ (2\mu z - x \sin \beta) \ dS^1}{\int \mu dS^1 \cos \beta} \quad . \quad . \quad . \quad (19)$$

To obtain the latter form we obtain the value of $Y_0 = W$ from equation $(136y)$, which gives

$$W = U_0^2 \int klm dS = U_0^2 K \int dS \cos \alpha \sin \alpha \cos \beta = K \int \mu U_0^2 \cos \beta dS^1$$

We may further simplify these results by taking the formulæ for a rectangle, which give, if c be the semi-span,

$$\int \sin \beta \ (2\mu z - x \sin \beta) \ dS^1 = S^1 \sin \beta \ (\mu c - x \sin \beta) \quad . \quad . \quad (20)$$

while the condition $M_q = 0$ gives

$$0 = \int (2\mu^2 z^2 - \mu x z \sin \beta) \ dS^1 = S^1 \left(\frac{2\mu c^2}{3} - \frac{xc}{2} \sin \beta \right) \quad . \quad . \quad (21)$$

Substituting in (19) we obtain

$$\tan \phi = \frac{U^2}{ag} + \frac{1}{3}\frac{U^2}{U_0^2}\frac{c}{a} \tan \beta \quad . \quad . \quad . \quad . \quad (22)$$

or if $U_0 = U$ (as is probably true)

$$a \tan \phi = 2 \text{ (velocity height)} + \tfrac{1}{3} c \tan \beta \quad . \quad . \quad . \quad (22a)$$

The conditions of perfect inherent controllability are now satisfied, and when the radius of the curve is large the angle ϕ is small. The effect of the air resistance is to slightly increase the angle ϕ required for equilibrium, thus producing a slight addition to the centrifugal force.

13. The objection to this particular form of solution is that it would hardly appear desirable to build an aeroplane satisfying the condition $B = C$. This would involve the assumption that the mean square of the distances of the elements of mass from the horizontal plane was as great as the mean square of their distances to the right and left of the plane of symmetry. Of course it might be *economically* possible to secure this condition by means of a large number of superposed surfaces instead of two at the outside, but this arrangement would have considerable inconveniences.

When $B > C$, as is the usual case, the condition $L_1 = 0$ can still be satisfied by varying the position of the turning point. This involves decreasing x by an

amount proportional to sin ϕ. But the result will be to effect a corresponding change in the value of M_q which will no longer be zero.

The difficulty could be got over by making

$$F = -(B - C)\ \mu \sec \beta \quad . \quad . \quad . \quad . \quad (23)$$

thus making the equations $M_1 = 0$ and $L_1 = 0$ equivalent. The objection to this has been pointed out, namely, that it would disturb the horizontal equilibrium by introducing a couple about Oz, and this couple would be large compared with the one about Oy which it is sought to eliminate.

But if the system is provided with a "neutral tail" for longitudinal steering and stability, and this tail is placed at a distance l behind the centre of gravity or $l - x$ behind the turning point, the tail will encounter a resistance $KS_2\ (l - x)\ U\Omega \sin \phi$, as may be seen from the formulæ of § 48, p. 76, and this resistance will have a moment about the axis of x equal to $KS_2\ (l - x)\ lU\Omega \sin \phi$ tending to tilt the tail up (*i.e.*, in the positive direction). As, however, the couple $F\Omega^2/g \sin \phi \cos \phi$ also acts in the same direction, it is clear that a negative value of F will only make things worse, and affect the longitudinal equilibrium more than it would be affected otherwise unless $l - x$ is negative, that is, unless the tail for longitudinal steering is in front of the turning point, and this condition is likely to be prejudicial to longitudinal stability.

It follows that it is best to make $F = 0$. Equilibrium can then only be maintained if pressure be exerted on the controlling rudders.

VIII.—EFFECT OF A RUDDER BEHIND THE CENTRE OF GRAVITY.

14. The effect of such a rudder will be to produce a force Z_1 and a couple M_1 connected by the relation $M_1 = lZ_1$, where l is the distance of the rudder behind the centre of gravity. These results are, of course, only approximate and conditional on the rudder being only turned through a small angle. If it is turned through a large angle it will also give rise to a resistance, which we do not here consider. The assumed position of the rudder is not such as to give any moment L about the axis of x.

The plan on which the following equations are built up is as follows:—

The extra moment in the L equation due to $B - C$ is balanced by shifting the turning point from its previous distance to a distance $x + \xi$ (although this makes ξ negative it is more convenient not to call the distance $x - \xi$).

This upsets the equilibrium in the M equation and the effect is balanced by the couple M_1 due to the rudder.

The corresponding force Z_1 is made to balance the other forces in the axis of z.

Finally, when the equations are written down we have three equations to determine the three unknowns ξ, Z_1 and ϕ corresponding with circular motion with any given values of the radius a and velocity U.

When these substitutions are made and we take account of the fact that L_q and M_q have been made to vanish for $\xi = 0$, we obtain

$$-W \cos \phi \frac{U^2}{ga} + W \sin \phi = -\frac{KU^2}{a} \cos \phi \left[\int \sin \beta\ (2\mu z - x \sin \beta)\ dS^1 \right.$$

$$\left. - \xi \int \sin^2 \beta dS^1 \right] - Z_1 \quad . \quad . \quad . \quad (24w)$$

$$(B - C) \sin\phi \cos\phi \frac{U^2}{ga^2} = -\frac{KU^2}{a} \cos\phi\xi \int z \cos\beta \sin\beta dS^1 \quad . \quad (24p)$$

$$0 = \frac{KU^2}{a} \cos\phi\xi \int \mu z \sin\beta dS^1 - Z_1 l \quad . \quad . \quad . \quad (24q)$$

and we have already seen from (20), (21), that for rectangular planes

$$\int (2\mu z - x \sin\beta) \sin\beta dS^1 = -\tfrac{1}{3}\mu c S^1 \sin\beta \quad . \quad . \quad (25)$$

Also we have, still for rectangular planes,

$$\frac{K\xi b}{2} S^1 \cos\beta \sin\beta = -\frac{(B - C) \sin\phi}{ga} \quad . \quad . \quad . \quad (26a)$$

$$Z_1 l = -(B - C) \sin\phi \cos\phi \frac{U^2}{ga^2} \frac{\mu}{\cos\beta} \quad . \quad . \quad . \quad (26b)$$

finally

$$-W \cos\phi \frac{U^2}{ga} + W \sin\phi = \frac{KU^2 S^1 \mu c}{3a} \sin\beta \cos\phi$$

$$+ (B - C) \sin\phi \cos\phi \frac{U^2}{ga^2} \left[\frac{\mu}{l \cos\beta} - \frac{2 \tan\beta}{c} \right] \quad . \quad . \quad . \quad (26c)$$

Substituting $W = KU^2 S^1 \mu \cos\beta$, which is true if U is equal to the velocity in rectilinear motion as assumed approximately; writing $U^2 = 2gh$, $B = Wr_2^2$, $C = Wr_3^2$, we get

$$-2h + a \tan\phi = \tfrac{1}{3} c \tan\beta + \frac{2 (r_2^2 - r_3^2) \sin\phi h}{a \cos\beta} \left[\frac{\mu}{l} - \frac{2 \sin\beta}{c} \right]$$

Since we have found from Mr. Harper's condition of stability that $\mu/l < \frac{3}{4} \sin\beta/c$ it follows that the last term must be negative and we may write the equation

$$a \tan\phi + \frac{2 (r_2^2 - r_3^2) h}{a \cos\beta} \left[\frac{2 \sin\beta}{c} - \frac{\mu}{l} \right] \sin\phi = 2h + \tfrac{1}{3} c \tan\beta \quad . \quad (27)$$

These results agree fairly well with what one would naturally expect from general considerations.

An alternative method is, of course, to use *ailerons*, the action of which can be made to produce an arbitrary couple, which couple will be denoted by L_1 in equation (5*p*) and will occur in all the equations of moments about the axis of x, such as (6*p*), (23), (24*p*).

IX.—CURVED WINGS.

15. The fact that it is impossible to obtain perfect inherent controllability in the previous case follows from (20), (21), which show that it is impossible to make M_q vanish without making Z_q negative. Although we have considered only the case of rectangular plane surfaces, a familiarity with integration formulæ and properties of moments of inertia will show that the same is necessarily true for wings of any shape provided that μ and $\sin\beta$ are everywhere constant, that is, that the wings are plane surfaces with a single dihedral angle between them. If, however, μz is proportional to $\sin\beta$ the integrals for Z_q and M_q are proportional so that if one vanishes the other vanishes too, as may be seen from (16*w*), (16*q*r) or (20), (21).

This leads to a very important conclusion, namely, that **curved wings may behave in a totally different way from a pair of straight planes with a single dihedral angle in the case of circling motion.**

If then, to go a little further, we make $\sin\beta/z$ increase as we go outwards towards the extremities of the wings, we shall be able to make Z_q positive, and thus to balance the term $-W\cos\phi a\Omega^2/g$ in condition (5w) which will therefore be satisfied by making $\phi=0$.

It thus appears possible with curved wings to design an aeroplane which will turn perfectly freely in any circle without canting over to one side.

The problem may be varied in many different ways, by taking not only β but also a, x and dS^1/dz to be functions of z. In this way it is possible to extend the calculations to wings of any shape whose curvature follows any assumed law. It would be an interesting exercise to perform the calculations with different assumed functional relations between the several variables and to trace the shapes of the wings so obtained. Such calculations would at least be as useful as the great majority of the papers which swell the pages of our mathematical journals and there should be no difficulty in finding cases where the integrations are simple.

By putting $h=U_0^2/2g$ and substituting from the conditions of equilibrium, we obtain for the equilibrium along the axis of z

$$2h\int\mu dS^1\cos\beta=\int\sin\beta\ (2\mu z-x\sin\beta)\ dS^1 \quad . \quad . \quad . \quad (28w)$$

The other equations to be satisfied are by (16)

$$0=\int(z\cos\beta-y\sin\beta)\ (2\mu z-x\sin\beta)\ dS^1 \quad . \quad . \quad . \quad (28p)$$

$$0=\int(\mu z-(x-x_0)\sin\beta)\ (2\mu z-x\sin\beta)\ dS^1 \quad . \quad . \quad (28q)$$

$$0=\int\mu\left\{(x-x_0)\cos\beta-\mu y\right\}dS^1 \quad . \quad . \quad . \quad . \quad . \quad (28u)$$

and in addition we have the geometrical relations

$$dzdx=mdS=dS\cos a\cos\beta=dS^1\cos\beta/\cos a$$

$$0=ldx+mdy+ndz \quad . \quad . \quad . \quad . \quad . \quad (29a)$$

$$\text{or } 0=\mu dx+dy\cos\beta+dz\sin\beta \quad . \quad . \quad . \quad . \quad (29b)$$

together with the usual formulæ $l=\sin a$, $m=\cos a\cos\beta$, $n=\cos a\sin\beta$ (29c)

$$\mu=\tan a,\ dS^1=dS\cos^2 a \quad . \quad . \quad . \quad . \quad (29d)$$

16. In the next place it is to be observed that no difficulty arises in reconciling (28u) and (28p) with the remaining equations of condition. If these conditions are not already satisfied it is only necessary to displace the wings relatively to the centre of gravity, thus adding arbitrary constants, say ξ, η, to the values of x, y, in these equations. It is possible even with narrow wings to assume that x is constant, so that x^1 or $x-x_0$ represents the displacement necessary for the purpose, *i.e.*, the constant ξ. If we suppose μ constant and multiply (28p) by μ and subtract from (28q) we get

$$\int(2\mu z\sin^2\tfrac{1}{2}\beta+(\mu y-x^1)\sin\beta)\ (2\mu z-x\sin\beta)\ dS^1=0 \quad . \quad (30a)$$

while (28u) may be written

$$\int\left\{(\mu y-x^1)+2x^1\sin^2\tfrac{1}{2}\beta\right\}\mu dS^1=0 \quad . \quad . \quad . \quad . \quad (30b)$$

X.—A TYPICAL TEST CASE.

17. In order to see how far these conditions are practicable, I consider a test case satisfying the following assumptions:—

α = constant, and therefore μ = constant
$\sin\beta$ proportional to z^n where n is positive
dS^1 the element of area proportional to dz

Under the latter condition $dS^1 \cos\beta/\cos\alpha$ will be the projection of the element on the plane of (x, z) and the assumption is that the *width* of this projection is proportional to $\cos\beta$, the wing thus narrowing towards the tip. Under these circumstances the limits of β may be 0 and 90° if desired, the wings thus curling up to the vertical position. If c_1, β_1, be the limits for z and β and we neglect $x - x_0$ or x^1, (28*w*), (28*q*), give

$$2h\int \mu dS^1 \cos\beta = \frac{2\mu c_1 \sin\beta_1 S^1}{n+2} - \frac{x \sin^2\beta_1 S^1}{2n+1}$$

$$0 = \frac{2\mu^2 c_1^2 S^1}{3} - \frac{x\mu c_1 \sin\beta_1 S^1}{n+2} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Substituting, we have

$$x = \frac{2\ (n+2)\ c_1}{3}\ \frac{\tan\alpha}{\sin\beta_1} \quad . \quad . \quad . \quad . \quad (32)$$

and remembering that $\int dS^1 \cos\beta_1$ is the projection of the wing area on the plane of xz, if we call this Σ, and call the actual area S^1, we get

$$h = \frac{S^1}{\Sigma}\ c_1 \sin\beta_1 \times \frac{-(n-1)^2}{3\ (n+2)\ (2n+1)} \quad . \quad . \quad (33)$$

This result shows that β_1 must be negative, in other words, that **the wings must be bent downwards towards the tips, not upwards, and that the turning point will be in front of and not behind the centre of gravity, thus closely resembling the action of gulls in circling flight.**

If $\beta_1 = \pi/2$ **it will also be seen that the wings will be pointed at their tips, and further, that since** $\tan\alpha$ **is small, the turning point is then very little in front of the centre of gravity.**

If instead of assuming dS^1 proportional to dz we assume dS^1/dz to be proportional to any power of z, say z^i, we get a very similar result, the fraction corresponding to (33) being

$$\frac{-(n-1)^2}{(3+i)\ (n+2+i)\ (2n+1+i)}$$

18. I have made several attempts to ascertain whether the conditions of the previous paragraphs can be satisfied by any system whose wings are bent or curved upwards, and these trials have led to the following.

XI.—THEOREM.—A System with Upturned Wings, none of which are in front of or behind the Centre of Gravity and the Angles of Attack of which are everywhere positive cannot possess perfect Inherent Controllability.

Multiply equation (28*w*) by x and (28*q*) by z and subtract, assuming x^1 or $x - x_0$ to be zero and we get

$$2hx\int \mu dS^1 \cos\beta = \int (-4\mu^2 z^2 + 4\mu z x \sin\beta - x^2 \sin^2\beta)\, dS^1$$

$$= -\int (2\mu z - x \sin\beta)^2 dS^1 \quad . \quad . \quad . \quad . \quad (34)$$

This shows that x must be negative or the turning point in front of the centre of gravity. But by (28q)

$$x\int \mu z \sin \beta dS^1 = \int 2\mu^2 z^2 dS^1 \quad . \quad . \quad . \quad . \quad (35)$$

Hence x cannot be negative unless $\sin\beta$ or μ is somewhere negative, which proves the proposition.

If the wings are of uniform width and μ is everywhere constant, $\sin\beta dS^1$ is proportional to $-dy$ so that the coefficient of $(-x)$ is proportional to $\int z dy$, that is, to the area in the plane of yz bounded above by the wing outline and below by the chord joining the wing tips. It is advantageous to make this area large, for which purpose the wings should be most bent down at the tips, or even in extreme case bent upwards near the centre so as to give a great bend down at the tips.

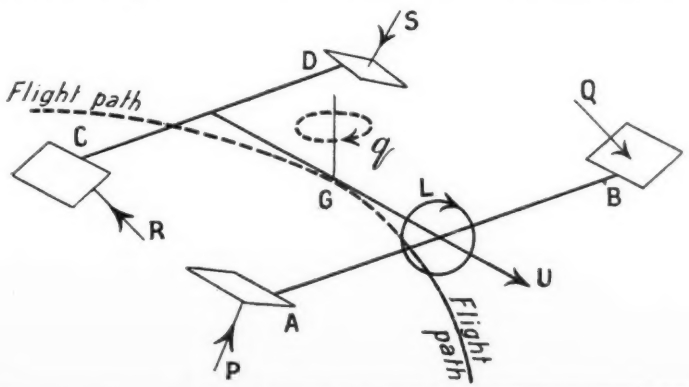
If the wings are not of uniform width it will be similarly seen that $\int z \sin\beta dS^1$ is proportional to the volume of the solid contained between the wing surface, and the cylindrical surface formed by horizontal chords joining points symmetrically situated on the boundaries of the pair of wings.

A possible way of securing the conditions here proved to be necessary for perfect inherent controllability would be by making $\beta = 0$ except near the tips of the wings and there making *either* β or μ negative. This leads to the case of the straight plane with terminal stabilisers which we proceed to discuss in Section XIII.

XII.—"FOUR-WINGED" OR TANDEM PLANES.

19. There is another entirely different way of obtaining "inherent controllability" in the sense here implied, by making the condition $M_q = 0$ independent of the position of the turning point, we leave the two variables x_0 and ϕ at our disposal to satisfy the two other equations of lateral equilibrium. It remains to be shown that systems can be devised possessing this property, although it will be found that a necessary condition is that some of the surfaces should be in front and some behind. Broad wings having a considerable measurement from front to back, such as are found in many birds as well as insects, are not to be regarded as excluded, except after fuller investigation than what is here contemplated.

The system here chosen for illustrating this possibility is one which I suggested some time ago in the AËRONAUTICAL JOURNAL* in a short note unaccom-



* April, 1911, page 46.

panied by evidence of a substantial character. It is taken as consisting of two systems of planes or wings arranged one behind the other, one being bent or curved upwards and the other downwards towards the extremities. From (28*q*) using x^1 to denote distances of planes in front of the centre of gravity, it follows that the necessary conditions for this kind of inherent controllability are given by the equations

$$0 = \int (\mu z - x^1 \sin \beta) \sin \beta dS^1 \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$\text{and } 0 = \int (\mu z - x^1 \sin \beta)\ (2\mu z - x^1 \sin \beta)\ dS^1 \quad . \quad . \quad . \quad (37)$$

taken over the sum total of the wing elements.

Now the first condition is satisfied by any pair of surfaces at equal distances in front of and behind the centre of gravity, one being bent upwards and the other downwards according to the same law. Thus, using suffixes to distinguish them, and supposing x_1, x_2, to be the distances from the centre of gravity taken positive when in front, we have at corresponding points of the two surfaces

$$c_1 = c_2,\ dS_1{}^1 = dS_2{}^1,\ \mu_1 = \mu_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

$$\beta_1 + \beta_2 = 0,\ x_1 + x_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

and from $dy = dz \tan \beta$ it follows that at all such corresponding points

$y_1 + y_2 =$ constant $= y$.

= distance of centre of symmetry of wing area below centre of gravity . . . . . (40)

With these symmetrical relations, (37) requires that

$$0 = \int (\mu_1 z_1 - x_1 \sin \beta_1)\ (2\mu_1 z_1 - x_1 \sin \beta_1)\ dS_1 \quad . \quad . \quad . \quad (37a)$$

this result covering both the rear and front planes.

20. If we try to satisfy these conditions by means of surfaces merely bent up at a single plane dihedral angle in the plane of symmetry (β and μ constant) and rectangular in shape, we obtain a quadratic equation for x_1, of which the roots are imaginary, showing that **with rectangular planes and simple plane dihedral angles the problem is impossible.**

The right hand side of (37*a*) is in fact essentially positive in this case. But as the integrand in (37*a*) is negative where $x \sin \beta$ lies between μz and $2\mu z$, it is clear that the present conditions allow considerable latitude both in the form of the wing surfaces and in the manner in which they are bent or curved.

As particular solutions we have only to assume that either

$$\mu z - x^1 \sin \beta = 0 \text{ or } 2\mu z - x^1 \sin \beta = 0 \quad . \quad . \quad . \quad (41)$$

at every point of the wing surfaces.

21. If we take the latter solution the remaining conditions of lateral equilibrium reduce to

$$2\ (r_2{}^2 - r_3{}^2)\ h \sin \phi = - a x_0 y \frac{\int dS^1 \sin^2 \beta U^2}{\int \mu dS^1 \cos \beta U_0{}^2} \quad . \quad . \quad . \quad (42p)$$

$$a \tan \phi = 2h - \frac{x_0 \int \sin^2 \beta dS^1}{\int \mu dS^1 \cos \beta} \frac{U^2}{U_0{}^2} \quad . \quad . \quad . \quad . \quad . \quad (42w)$$

which lead to

$$a \tan \phi = 2h \left\{ 1 + \frac{r_2{}^2 - r_3{}^2}{ay} \sin \phi \right\} \quad . \quad . \quad . \quad . \quad . \quad (42u)$$

The equation of equilibrium (28u) gives, however,

$$\int \mu^2 y dS^1 = \int \mu_1 x_1 \cos\beta_1 dS_1 + \mu x_2 \cos\beta_2 dS_2$$

$$= \int \mu\delta \cos\beta_1 dS_1 \quad . \quad . \quad . \quad . \quad . \quad (42u)$$

taken over one wing with δ equal to the difference between the distances of the fore and aft wings from the vertical through the centre of gravity. This means that $x_1 + x_2$ is not exactly zero, so that the solution here obtained is only approximate. Further, for longitudinal stability it is important that μ_1 should be $> \mu_2$. In any case the present solution does not quite meet all requirements. But since we have seen that the arrangements of the wings can in general be varied within sufficiently wide limits without affecting the conditions of the problem, it certainly appears possible to obtain a slightly different solution not open to these objections.

22. The other solution

$$\mu z - x^1 \sin\beta = 0$$

does not present this difficulty, for if the front and rear planes everywhere satisfy this condition, both (36) and (37) will vanish identically. In this case equations (39) are unnecessary, and no relation need connect the constants of the front and rear planes barring any conditions derived from the longitudinal equilibrium and stability or considered desirable from a longitudinal consideration. With this solution $x_0 \int dS^1 \sin^2\beta$ is replaced by $\int (x_0 - x^1) dS^1 \sin^2\beta$ in (42w) and $yx_0 \int \sin^2\beta dS^1$ by $\int (x_0 - x^1) \sin\beta (y \sin\beta - z \cos\beta) dS^1$ in (42p).

The geometric interpretations of the various integrals are very simple if the wings are strips of constant width, x^1 is constant and μ constant. We have $\int x^1 \sin^2\beta dS^1$ proportional to $\int \mu z \sin\beta dS^1$ or to $\int z dy$, the area of the vertical projection of the wing curve bounded by the chord. Under the same circumstances $\int x^1 y \sin^2\beta dS^1$ is proportional to $\int \mu z y dy$, the product of this area into the distance from the axis of y of the centroid or the part on the positive side of the axis of y, and $\int x^1 \sin\beta z \cos\beta dS^1$ is proportional to $\int \mu z^2 dz$ or $\mu c^3/3$.

23. It will be observed that condition (36) is identical with the condition $M_w = 0$ (compare p. 154, equations 169) as it should be, for a displacement of the turning point forwards or backwards is equivalent to impressing a corresponding velocity component w along the axis of z. Systems satisfying this condition thus possess the further advantage that **if suddenly struck by a side gust of wind no couple tending to produce rotation about the axis of y is set up.**

XIII.—STRAIGHT PLANES WITH TERMINAL STABILISERS.

24. The formulæ are somewhat simplified in the case of the system of § 96 in which the main lifting surface S is straight, but carries at its extremities a pair of "stabilisers" or surfaces each of area $\frac{1}{2}T$, bent up at a positive or negative angle β and having the angle of attack α^1 whose tangent is μ^1. With the further notation and explanations of § 96, we have the following expressions for the forces and couples:—

$$qZ_q = \frac{KU^2}{a} \cos\phi \sin\beta \ (2\mu^1 z - x \sin\beta) \ T^1 \quad . \quad . \quad . \quad . \quad . \quad (43w)$$

$$qL_q = \frac{KU^2}{a} \cos\phi \ \{ -2\mu I^1 - (2\mu^1 z - x \sin\beta) \ (\cos\beta - y \sin\beta) \ T^1 \} \quad (43p)$$

$$qM_q = \frac{KU^2}{a} \cos\phi \ \{ 2\mu^2 I^1 + (2\mu^1 z - x \sin\beta) \ (\mu^1 z - x^1 \sin\beta) \ T^1 \} \quad (43q)$$

where x, x^1, as usual represent the distances of T^1 in front of the turning point

and centre of gravity respectively. The equation of moments about the axis of z in rectilinear motion requires that

$$(\mu S^1 + \mu^1 T^1 \cos\beta)\, x^1 - (\mu^2 S^1 + \mu^{12} T^1)\; y = 0 \qquad . \quad . \quad (43n)$$

25. The conditions $L_q = 0$, $M_q = 0$, will be consistent if

$$\mu^1 z - x^1 \sin\beta = \mu\,(z\cos\beta - y\sin\beta)$$
$$= \mu\zeta \qquad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

If $x^1 = 0$, this reduces to $\mu^1 z = \mu\zeta$ and is the condition for neutral or critical equilibrium obtained by Mr. Harper (p. 156, equation (172)) stability requiring that $\mu\zeta - \mu z^1$ shall be positive. As Mr. Harper's investigation does not include a rudder plane it is clear that an addition of this kind, if suitably situated, could change the critical into stable equilibrium besides its surface acting as a guide in circling flight. The condition can evidently only be satisfied (if x^1 is small or ζ positive) by making $2\mu^1 z - x\sin\beta$ negative.

The further conditions that the aeroplane may circle freely without canting over require that Z_q should be positive, and we thus see from $(43n)$ that $\sin\beta$ must be negative or the wings bent down in order that this may be possible. Further, $x\sin\beta$ must be positive so that x is negative and the turning point in front of the centre of gravity, as previously proved generally in Section X. of this paper.

26. The condition that in a four winged or tandem system M_q shall be independent of x_0, or as we have seen $M_w = 0$ becomes, with suffixes for the two sets of stabilisers,

$$T_1^{\,1}(\mu_1 z_1 - x_1 \sin\beta_1)\,\sin\beta_1 + T_2^{\,1}\,(\mu_2 z_2 - x_2\sin\beta_2)\,\sin\beta_2 = 0 \qquad (45)$$

This condition is satisfied by $T_1^{\,1} = T_2^{\,1}$, $x_1 + x_2 = 0$, $\beta_1 + \beta_2 = 0$, as postulated in (38), (39), but it may also be satisfied in other ways as well, as it contains ten variables. Moreover, this condition is independent of the arrangement of the lifting planes. There is, therefore, no difficulty in adjusting the areas, co-ordinates and inclinations of the lifting planes, so as to satisfy the conditions of equilibrium corresponding to $(43u)$ and at the same time make $\mu_1 > \mu_2$ in order to satisfy the conditions for longitudinal stability. The condition of equilibrium is written down by duplicating the left hand side of $(43u)$ with separate suffixes for front and rear planes.

It may be useful to call attention to the fact that in the remaining condition which we may write

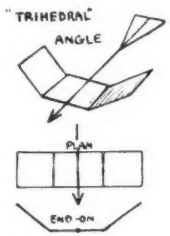
$$\Sigma\,\left\{2\mu^2 I^1 + (2\mu^1 z - x^1\sin\beta)\,(\mu^1 z - x^1\sin\beta)\,T^1\right\} = 0$$

where the Σ refers to the sum of the terms for the front and rear wings, the term containing T^1 has its largest negative value when $x^1\sin\beta = \frac{3}{2}\mu^1 z$ and in this case it is only equal to $\frac{1}{4}(\mu^1)^2 z^2 T^1$ while $2\mu^2 I^1$ becomes equal to $\frac{9}{8}\mu^2 z^2 S^1$ if the main planes are rectangular. It will thus be seen that it is *difficult* by means of stabilisers alone to realise the conditions of inherent controllability of the four-wing system here considered. Of course, this can be done by making μ^1 considerable, and as $\mu^{12}T^1$ is equal to $T\sin^2\alpha$ it will be seen that this may be just possible when α is small. At the same time it will be evident, from what has been shown, that it is better to employ curved planes than stabilisers for this particular object.

The fact that certain important terms in our formulæ have been found under certain circumstances to be proportional to the area of the projection of the wing curve on the plane of yz indicates that a wing sharply bent near the tips may be more efficient than one bent in a continuous curve, and the area in question can be made nearly double as great as if the wings were bent into a plane dihedral angle at their base. Thus, a pair of stabilisers may be considerably more efficient in securing controllability than the conventional dihedral angle.

XIV.—TRIHEDRAL ANGLES.

27. We may use the term "trihedral angle" to denote a lifting surface which is straight or horizontal in the centre and is bent up (or down) at a constant angle β for a finite distance towards its extremities, there being thus one bend on each of the two wings. The bent up parts will thus differ from the "stabilisers" of the last section merely in that they extend for a considerable distance in the direction of the axis of z.



By using the well known properties of moments of inertia formulæ ($43w$, p, q) can be extended to these cases with the following modifications:—

(1) Where first powers of z or y occur their values are to be taken to be those at the centroid of the area.

(2) Where T_2 occurs in ($43q$) we must add to z^2 the square of the radius of gyration of the bent up portion about an axis through its centroid parallel to the axis of y. If this is called k_2^2 it is meant that an extra term $2(\mu^1)^2 k_2^2 T^1$ must be added to $2\mu^2 I^1$ in ($43q$).

(3) In view of the fact that $y \cos\beta + z \cos\beta$ is constant along the bent up part, the effect in ($43p$) is represented by inserting an additional term after $-2\mu I^1$ of $-2\mu^1 k_2^2 T^1 \sec\beta$.

(4) There is no change in ($43w$).

28. It appeared to me to be desirable to ascertain whether it would be possible to substitute trihedral angles for the curved up and down surfaces of the four winged systems of Section XII. and thus obtain the kind of inherent controllability there described. It will be seen that this possibility is conditional on being able to make the integral

$$\int (2\mu z - x^1 \sin\beta)\ (\mu z - x^1 \sin\beta)\ dS$$

zero or negative.

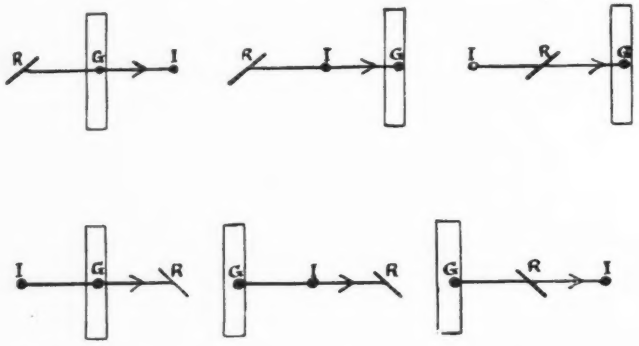
Now I find on working out the details (if my algebra is correct) that if the breadth of the surface is everywhere constant and μ is also everywhere constant, it is *just* possible, and only just possible, by suitably choosing the variables and arranging the bend at a particular place, to make the integral equal to zero, and in this case the bends will be at the point of trisection of the total span. (It is easy to make mistakes and get a different result.) This solution is only consistent with the symmetrical relations of (39) which do not, as we have seen, fit in with the conditions of longitudinal equilibrium and stability. Under all other circumstances the integral is positive and it follows that **surfaces of uniform breadth bent into a trihedral angle of the kind here described and with the angle of attack everywhere constant cannot be used for the purposes here considered.**

If the breadth of the surface be variable this difficulty disappears both for a trihedral or a dihedral angle. For example, having regard to the integrand of equations like (37*a*) we see that if the breadth of the wing surface increases from $z=0$ to $z=\frac{3}{4}x \sin \beta/\mu$ and decreases coming to the tip at $z=x \sin \beta/\mu$ we have a shape somewhat resembling a fly's wing, which is readily adaptable to the conditions of the problem. The form must in general be broader towards the tips than near the base.

XV.—ON THE BEST POSITION FOR THE RUDDER.

29. As an inherently controllable aeroplane is from the nature of the case deficient in lateral stability, the question of placing the rudder plane in such a position as to render the machine stable when used as a fin cannot be definitely decided without having recourse to the methods described in "Stability in Aviation." At the same time there are several general considerations bearing on the matter which may not improbably lead to the right results.

SIX POSSIBLE RELATIVE POSITIONS OF RUDDER, TURNING-CENTRE, & C/G.



Suppose the aeroplane to be in rectilinear motion and to receive a small angular rotation about a turning point I, this being the limiting position of the turning point for circular motion with a large radius for $\phi=0$. Call G the centre of gravity, R the position of the rudder, regarded as a fin. Then it will be found that—

If G is between R and I the resistance of the rudder tends to retard both the translational and rotational components of the added motion, which fact suggests probable stability.

If I is between R and G the resistance tends to decrease the angular but to increase the lateral component, thus tending to shift I further away.

If R is between G and I the resistance tends to decrease the lateral, but increase the angular component, bring I nearer to R and causing the machine to curl up in a spiral.

The third alternative seems to indicate evident instability, and even the second seems very doubtful from the point of view of stability. I should infer that when the turning point is behind the centre of gravity the rudder should be in front, and when in front the rudder should be behind. If the turning point ever coincides with the rudder the latter will be ineffectual in maintaining steady circular motion, and it looks as if the machine might become uncontrollable.

As regards the actual position of the rudder, it is probable that the best results will be got when R and I are related in position like the centres of

suspension and oscillation in a compound pendulum, so that $GI.GR = -r_2^2$ the square of the radius of gyration of the mass of the aeroplane about the axis of y.

In treating this as a question of rigid dynamics, pure and simple, we are guided by the same considerations as are assumed in the theory of initial motions, and we for the time leave out all consideration of the air resistances because the changes in these depend on the changes of velocity, which are initially zero, whereas the action of the rudder determines the components of initial acceleration which are finite. This consideration leaves very little doubt about the matter. An examination of some of the problems on initial motion in Routh's Rigid Dynamics will make this clear.

XVI.—SUMMARY AND CONCLUSIONS.

1. In steady motion in a horizontal circle, both the longitudinal and the lateral equations of equilibrium are affected.

2. The turning point may be in front of or behind the centre of gravity, its distance when in front being denoted here by c.

The axis of the aeroplane then envelopes a circle of a certain radius a, the real radius of the circle described being $\sqrt{(a^2 + c^2)}$.

The system usually cants over sideways through a certain angle ϕ.

3. Given the velocity and radius of the circle it is not usually possible to satisfy the three equations of lateral equilibrium by assigning suitable values to c and ϕ, but when this is possible the system is said to be inherently controllable.

In an inherently controllable system the rudder planes merely act as guides and it is necessary that they should be so placed as to render the motion laterally stable.

In other cases steady motion can only be maintained by pressure exerted by the rudders or a couple applied by means of *ailerons* or some such action representing the third unknown variable required for the solution of the three simultaneous equations of lateral equilibrium.

4. In a system of straight planes $\sin\phi$ is proportional to the radius a of the envelope, but it also appears that the other conditions of lateral equilibrium are only possible when pressure is applied by means of a rudder and when a and ϕ have certain definite values. The only way of varying the radius of the circle actually described is by varying the position of the turning point which may be in front of or behind the centre of gravity.

The addition of boxed in ends or vertical partitions improves the steering, but it still leaves $\sin\phi$ proportional to a. The inference one would naturally derive from the formulæ is that all such systems would be liable to sway from side to side of the straight path in curved arcs of finite radius. In no case can the radius of the circular envelope exceed the limit corresponding to $\phi = 90°$.

5. With bent up wings, as in the "Antoinette type," it is possible to satisfy the conditions of equilibrium so that a is no longer limited and ϕ no longer large. Such a system can be steered in a circle of large radius without being inclined at a large angle.

In general, circular motion can only be maintained when pressure is applied by means of a rudder or a couple applied by means of *ailerons*, but if the two principal moments of inertia about axes perpendicular to the line of flight are equal, the rudder exerts no pressure and the system is inherently controllable, the inclination satisfying the relation $U^2 = ga \tan\phi$.

6. Another kind of "inherent controllability" in which the system always remains level, the inclination ϕ being zero, is possible in certain systems. A

necessary condition is that the wings should be bent downwards and not upwards at the tips, and it will be usually advantageous that they should be most bent down at their extremities. The condition representing this fact is that the space between the wings and a chord joining their tips should be as large as possible.

This arrangement of the wings somewhat reproduces the action of gulls' wings in circling flight, and it will be found that differences in the form and curvature of the wings may have a considerable influence in the problems of this class.

7. A third kind of "inherent controllability" is only possible when portions of the wing surface are in front of or behind the rest; and a possible solution exists in the form of a system suggested by me in the AËRONAUTICAL JOURNAL* with front and rear planes, one set being turned upwards and the other downwards.

It appears, however, from the analysis that the necessary conditions cannot be satisfied in the case of surfaces of uniform breadth bent up into a plane dihedral angle at the centre or bent into a trihedral angle at some points intermediate between the centre and tips. They can, however, be readily satisfied by *suitably* curving the wings or by varying their shape so as to make them as a rule broader towards the tips than near the base. The present arrangement has the further advantage that the system would not tend to turn round sideways if struck by a side gust of wind, and I should consider it worth trying experimentally.

8. Although no attempt has been made to discuss the analytical conditions of inherent stability further than has been done in "Stability in Aviation," it appears from general considerations that the rudder plane at least in an inherently controllable system should be placed on the opposite side of the centre of gravity to the turning point, and that difficulties, probably instability, must necessarily occur if the rudder is between the centre of gravity and the turning point. It seems almost certain that the best position for the rudder is when it and the turning point are in the relative positions of the centres of suspension and oscillations of the system when treated as a compound pendulum.

PROBLEMS.

1. A detailed examination of the conditions of longitudinal equilibrium in the case of steady circular motion, and of their effects on the conditions of lateral equilibrium.

2. Further discussion of the systems of Section IX.-XI. with special reference to the height of the centre of gravity and the equation of moments about the axis of x; in particular complete solutions of the simultaneous equations of Section IX. for a test case such as that of Section X., employing approximations where desirable.

3. Similar complete solutions for four-winged systems satisfying the properties of Section XII., also for lifting planes with stabilisers where these are effective.

4. Application of the analytical methods of my book to investigate the lateral stability of such of the present systems as are not there discussed, and the positions of the rudder consistent with stability.

It will be observed that these problems mainly involve the working out of algebraical details, and to anyone who likes this kind of work they should present little difficulty. I think they may lead to results which it may be desirable to know, as these may affect the practicability of some of the systems here considered. Of the desirability of such detailed discussions sufficient evidence is shown in this paper, the differences between the behaviour of bent and curved wings not being such as could have been or were anticipated when the present investigation was commenced.

* April, 1911, page 46.

The CHAIRMAN: In rising to propose a vote of thanks to Professor Bryan I am sure that it is not necessary to remind those present that the lecture to which it has just been their privilege to listen is the third of the annual lectures founded in commemoration of the late Mr. Wilbur Wright.

It is fitting and appropriate that on the present occasion a few words should be said on the work done by Mr. Wilbur Wright, and more broadly by the Wright Brothers, and on the place which it is anticipated will be given to their work in the history of aeronautical development, when sufficient time has elapsed to allow the work of the Wrights and their contemporaries to be viewed in their true perspective.

At the outset I will make a statement, which, though it may not find universal acceptance at the moment, will, I firmly believe, at some future time be looked upon as fairly representing the truth; I do not think the position of the Brothers Wright as pioneers in the development of aeronautics will be found to rest so much on any scientific work they may have accomplished, but rather upon their achievement as pioneers of the art of flight, of which they were certainly the most distinguished exponents of their time.

I do not wish it to be thought that I am neglecting the work done by the Wright Brothers, from a quasi-engineering point of view, as pioneers and constructors and designers of their own machines, either full scale gliders or as finally represented in their completed aeroplane; in this work alone there is evidence not only of real knowledge but also of great skill and daring, attributes which are nearly always to be found in combination where success in any new problem is achieved. To be just to the Wright Brothers, however, and to be fair both to those who preceded and followed them, and to those whose achievement in the same field of work was independent of their own, it is necessary to appreciate that the great achievement of the Wrights must be regarded as centred in *actual performance*.

The problem of flight was simplified by the Brothers Wright into finding a solution to the aerodynamic side of the problem. There were others who talked of stability and who studied stability and believed some kind of inherent or automatic stability was necessary. The Brothers Wright had the acumen to perceive that to attempt too much was to court failure, and, following to some extent the teaching of Lilienthal, determined to treat this side of the problem as an affair of the *art of flight* and so simplify the issue and concentrate on the really essential, the problems of *sustentation, motive power,* and *propulsion,* for the solution of which the material was already almost within reach.

Thus, in the early Wright machine there was neither longitudinal nor lateral stability, but the whole machine was worked out to be as completely under the control—positive control—of the aeronaut as possible, and though in the course of their experiments I understand that the Brothers Wright made several trials of varying the load distribution and flight organs to obtain stability, their conclusion in this direction was that in the then state of the art the purely hand controlled machine was the best. It is thus that the *Art of Flight* became so important and distinctive a feature of the Wrights' work.

I had many opportunities of seeing Wilbur Wright and talking with him at the time when he was demonstrating his machine in France in 1908, and then formed the opinion that the characteristic in which Wilbur Wright excelled was in skill in the art of flying. One has only to recall his performances in France to see how much superior he was to the other fliers at that time. When Farman was flying cautiously round in circles of large radius, Wilbur Wright, by virtue of his combined rudder and warping control, was able to turn and twist in the air in curves of very small radius, in brief, his flying was like that of a bird.

I always found Wilbur Wright commendably careful of the reputation of flying from the public standpoint; he had a great sense of his personal responsi-

bility in this regard. There is no doubt that if he had wished he could have undertaken cross country flights with good prospect of success long before anything of the kind was attempted, but he possessed the moral courage not to risk the reputation of the new mode of locomotion for the sake of personal popularity.

The future of the flying machine is undoubtedly wrapt up in the fuller solution of the problems of stability. The aerodynamic problem is of course always with us—the continually increasing flight speed with the corresponding exigencies in the matter of h.p.—but this is a matter to-day of ordinary engineering development. The completely stable machine, however, is a thing of quite recent achievement and there yet remains much to be done.

It is in the steps constituting the foundation of this work that the theoretical investigations so ably carried out by Professor Bryan have already proved to be of great value and we may look to still further assistance from his more recent work, an example of which we have had the pleasure of hearing expounded to-night.

Professor Bryan has honoured me by referring to certain of my work as touching on the problem of stability. It is a fact not without interest that though my own methods and those of Professor Bryan have but little in common, whenever they do come into contact—*i.e.*, when they give results which are directly capable of comparison, the agreement is quite remarkable. To understand the work of Professor Bryan as it should be understood, one ought to devote fully twelve months to the study of Routh's Rigid Dynamics; in this I confess myself at a disadvantage, hence my knowledge of Professor Bryan's work is by no means as intimate as I should wish; it is all the more gratifying under these conditions to find from a comparison of results that we are so much in agreement.

The present occasion is one on which I may perhaps be permitted to offer my congratulations to the Aëronautical Society on its prosperity. I might almost say (in view of the vicissitudes through which the Society has passed) *on its existence*. It is certainly something of which any Englishman may be proud that this Society, the first of its kind to have been established in any part of the globe, will next year record (and we will hope duly celebrate) its 50th anniversary. It is not easy to-day to over appreciate the enthusiasm and prophetic foresight which led in 1866 to the founding of a Society destined only to see its objects fully achieved some 40 years later, and now, when that which we may fairly call the *life's work* of the Society has been accomplished, and the foresight of its founders has been more than vindicated, it is gratifying to find that the Society has found a fitting sphere of activity in awarding and giving appropriate recognition to those who have assisted in the fight. Such recognition is especially to be valued when bestowed upon those whose lot has been to work behind the scenes rather than in the limelight of publicity, those whose work like that of the Lecturer, Professor Bryan, is none the less of value because of its abstract and recondite character. (Loud applause.)

Major-General R. M. Ruck, C.B.: I rise to propose a vote of thanks to Mr. Lanchester for presiding here to-night. The Society is very much indebted to him and greatly appreciates his presence in the chair at this lecture and the remarks he has made.

Mr. Lanchester's distinguished work in connection with the theory of flight is so well known that I need scarcely allude to it, but we can congratulate him on his far-seeing conception of the possibilities of flying long before it was an accomplished fact. All connected with aeronautics should be deeply grateful to Mr. Lanchester for his work, which has, like that of Professor Bryan and Mr. Busk, assumed national importance.

On this point Colonel Brancker, the present head of the Air Department at

the War Office, has kindly consented to give you further information, and I take advantage of this occasion to assure him how greatly this Society, and I am sure all here, have appreciated the bravery, skill and devotion shown by the Flying Corps of both Services. Nothing could have been finer, nothing more useful to the country in the present crisis. (Loud applause.)

Colonel W. S. BRANCKER (Deputy Director of Military Aeronautics): I rise to second Major-General Ruck's proposal of a vote of thanks to our Chairman, and I take this opportunity of making a few brief remarks to show of what great value mathematics and scientific research has been to the Royal Flying Corps.

The present war has probably fulfilled some of Wilbur Wright's wildest dreams; and I think that the work of the Flying Corps has gone to prove that we as a nation have nothing to reproach ourselves with in the use we have made of Wilbur Wright's wonderful heritage.

We took the field with many different types of aeroplanes. Two months of war proved that the aero-dynamical qualities and the construction of the British types were undoubtedly superior to both those of our Allies and our enemies. This fact, backed by the incomparable courage of our pilots, enabled us to establish a virtual command of the air in the zone occupied by the British Army. Every German pilot who made his appearance was attacked, and it became an invariable rule that no German would face a British aeroplane. Lately, however, the German aeroplanes have greatly improved in their speed and climb, I think probably owing to the introduction of a more powerful and more efficient engine. At the moment, there are German aeroplanes which can out-pace all but our fastest machines, with the result that the hostile pilots have become bolder in the aerial combat and now have to be driven off instead of hunted. The Germans have given up reconnaissance over our lines to a great extent because we have given them such a bad time, but they have lately devoted themselves to attacking our aeroplanes while they are observing artillery or making a reconnaissance. It has consequently become almost a normal custom to send up two aeroplanes together, one to do the reconnaissance and the other the fighting.

The reason for this German advance is not our neglect of the science of aerodynamics; it is the outcome of the fact that at the outbreak of war the aeronautical engine trade in England was practically undeveloped, and up till quite lately we had depended very largely on comparatively low-powered French engines. Already, however, higher powered British-made engines have made their appearance at the front and the pendulum is swinging back again; British design backed by sufficient horse-power is once more proving its superiority, and we have given the Germans some very ugly surprises lately.

Many points of aerodynamical interest have been brought out by the war of which I cannot speak now; the most interesting is, perhaps, the conflict between the necessity of carrying weight and the desirability of a rapid climb. The development of fighting in the air makes great demands in weight carrying qualities; practically every machine which takes the air now is equipped with some form of offensive firearm, or with bombs, over and beyond the endless accessories such as field-glasses, cameras, wireless instruments, etc.; and naturally the "pusher" type, which was the type that Wilbur Wright evolved, although comparatively slower, has proved its superiority for the use of weapons. Armour is also necessary to ward off the splinters of the anti-aircraft shell. At the same time speed, climb, and easy handling must be retained if the pilot is to have a fair chance of accounting for his enemy.

In spite of the demands of war for standardisation and rapid output, we are still developing and improving the aerodynamical qualities of the British aeroplane; as the war goes on the demands for speed, climb, and lift, will become

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more and more exacting, and I trust that we shall be able to meet them with the assistance of mathematicians like the lecturer to-night and the scientific research of the National Physical Laboratory. The stable aeroplane, of which I think Colonel Seely spoke to you on this same occasion last year, has become a standard type. This stable aeroplane was, I believe, proved mathematically possible in the first instance by Professor Bryan, whom we have heard lecture to-night; the model researches were carried out by the National Physical Laboratory; the full-size experiments and the first machine was made by the Royal Aircraft Factory; and it was first flown without controls by my departed friend, Mr. Busk, who was then in charge of the physical research and test work at the factory. A very efficient combination. (Applause.)

Stability has proved of very great value in war; the pilot, flying by himself, can use his hands freely for weapons of offence and defence and can even kneel on his seat and look over his tail during a reconnaissance. It is also the only type of machine on which a hastily trained pilot can be depended on to fly efficiently. We have to train pilots rapidly now, and the stable machine is an absolute godsend.

I think that we can say that Wilbur Wright and his genius have stood us in good stead in this great crisis. Personally, I trust that the next anniversary of this occasion we may be discussing the wonderful uses of aviation as a peaceful pursuit and not as a weapon of war. (Loud applause.)

The meeting terminated at 10.30 p.m.

OBITUARY.

SIR CHARLES SEELY, BART.

We deeply regret to record the death of Sir Charles Seely, Bart., at Brook House, Isle of Wight, on the 16th April, aged 81, from heart trouble.

Sir Charles Seely joined the Society in 1908. As he never took any active part in aeronautics his support of this Society must be accounted one of those kindly and generous actions for which he was noteworthy. His eldest son, Mr. Charles Hilton Seely, formerly M.P. for Lincoln, and also a member of this Society, succeeded to the baronetcy and estates.

We deeply regret to have to record the deaths of the following members on active service:—

LIEUTENANT-COMMANDER C. G. W. HEAD, R.N.

Clement Gordon Wakefield Head, Lieut.-Com., R.N., youngest and only surviving son of Mr. and Mrs. J. J. Head, of Seaford, Sussex, was lost in his submarine on the 1st December, 1914, aged 29. He had been a member of the Society since 1909, and would undoubtedly have won in the Air Service, had he chosen to leave the Submarine Service, an equally high reputation to that which he had gained in the latter Service.

SECOND LIEUTENANT S. P. COCKERELL, R.F.C.

Samuel Pepys Cockerell, Second Lieutenant, Royal Flying Corps, Military Wing, died in Egypt on the 20th March, 1915, suddenly, of acute smallpox, aged 34. Lieutenant Cockerell joined the Society in 1911, the year in which he took his pilot's certificate. He joined the R.F.C. in January, 1913, and was appointed to active service soon after the outbreak of war.

CAPTAIN A. G. FOX, R.E.

Alan Geoffrey Fox, Captain, R.E., aged 27, only surviving son of Mr. and Mrs. Charles Fox, 30, Ladbrooke Gardens, W., died of wounds sustained whilst carrying out an important bomb attack behind the German lines, on 9th May, 1915.

Captain Fox joined the Society in 1911, when in the old Air Battalion, and played an important part in the development of the Royal Flying Corps from that embryo service. He was mentioned in despatches for his work in the present campaign.

MAJOR B. H. BARRINGTON-KENNETT.

Basil Herbert Barrington-Kennett, Major, 2nd Grenadier Guards, was shot while leading his men against the Germans on 18th May, 1915.

Major Barrington-Kennett joined the Society in 1911. Like Captain Fox, he played an important part in the development of the Royal Flying Corps from the old Air Battalion. In 1912 he won the Mortimer Singer prize for the longest flight by an Army officer, by flying with a passenger, on a Nieuport monoplane, a distance of 249½ miles. For some time he was Adjutant of the R.F.C. at Farnborough, in which capacity his singular personal charm stood him in good stead. When war broke out he was appointed to the Staff as Deputy Assistant Adjutant and Quartermaster-General, subsequently leaving the R.F.C. to serve with his regiment, the Grenadier Guards. Major Barrington-Kennett married a few years ago. Mrs. Barrington-Kennett, to whom all sympathy will be extended, has done much valuable work on the Flying Corps Aid Committee.

REVIEWS.

The Zeppelin Raid in West Norfolk. Holcombe Ingleby, M.P. (London: Edward Arnold. 1915. pp. 31. Price 3d.)

The objects of this pamphlet are not aeronautical, but there are one or two points mentioned which are likely to interest the student of aeronautics. The main purport of the author is to prove that the Zeppelin raid on Norfolk was much facilitated by the help of German agents in this country, who by means of signals with motor car headlights, directed the course of the airship when over this country. From an aeronautical point of view the chief point is concerned with the visibility at night of aircraft from the ground and of the ground from aircraft. Mr. Ingleby finds it difficult to believe that it is possible to make out with quite reasonable distinctness, even on a night when there is no moon, the general features of the ground over which one is flying. The experience of many aeronauts, however, makes it clear that such is often the case. It does not, however, prove that the crew of the Zeppelin in question could know where they were striking the coast of England after a long night's journey over the sea, and this brings us to the second point, which is the question of setting one's course at night time. This matter was ventilated in connection with the proposed Trans-Atlantic trip and was considered by some to present no difficulty if flares were used to enable drift to be computed. To judge, however, from some of the evolutions of Zeppelins which reached this coast, the task was of much greater difficulty than some would have admitted, and the raiders might have been left groping in much more uncertainty if misguided public discussions of the exact course followed by them had not been placed at their disposal by the Press in this country, after each raid, until this method of assisting the enemy was forbidden.

Aircraft in the Great War. C. Grahame-White and Harry Harper. (London: T. Fisher Unwin. 1915. pp. 340. Illus. Price 7s. 6d.)

From the popular point of view this book, which is practically a bound newspaper, justifies its publication. If ever, however, someone arises, after the conflict, to write the technical treatise that should be written on Aircraft in the Great War, he will not be pleased to find the obvious title already appropriated.

The book is mainly filled with extracts or paraphrases of newspaper reports of the chief performances of aircraft that had been reported up to the beginning of this year. There are, however, informative little chapters intended for the lay reader unfamiliar with aeroplanes. Altogether the lay reader of this book will gain a very fair picture, although a somewhat theatrical one, of the accomplishments of aircraft in the war up to the date mentioned, and a useful indication of the manner in which their uses are likely to increase. The easy facility with which the subject is handled carries one along so smoothly that one does not, while reading, give a thought to the much more complete book that will have to be written after the war, with much the same title as this one.

Motion of Liquids. Lieutenant-Colonel R. de Villamil, R.E. (ret.). (London: E. and F. N. Spon. 1914. pp. 210. 86 illus. 30 tables.)

It would be useless to warn the busy or intolerant person not to read this book; the busy man would probably not see the advice, and the intolerant man is usually unknown as such to himself. Nevertheless, this is distinctly a book for leisurely study, aided by a painstaking desire to see the author's point of view.

XUM

The chief aim of the book is to bring out the difference between a " static " and a " non-static " liquid, in other words, the difference in resistance that is caused by a liquid flowing past a body at rest, as compared with the resistance of the same body moving through the liquid at rest. This appears at first sight to impugn the principle of relativity; the author, however, is fully alive to this point, and in reality, so far from impugning that principle, is merely endeavouring to make clearer the conditions in which it is not applicable. To say that he puts his case so effectively as to meet all objections would be to go too far, there is still much to be said on the subject. The evidence collected by Colonel de Villamil, especially his Resumé of the Experiments of Dubuat and Duchemin, is very valuable, and deserves careful consideration. These experiments show that in many cases, where ordinarily one would say that it would be easy to reverse an experiment in which a liquid flows past a body or *vice-versâ*, a different numerical value has been obtained for the resistance caused by the relative movement of fluid and body, thus making clear that the conditions cannot have been reversed in reality.

To make a full examination of this book would require many pages. Actually to appreciate the points brought out one must have studied the large number of works referred to by the author, and have had leisure to digest them. If the reader has sufficient leisure it would be difficult after reading Colonel de Villamil's book not to want to go very much more deeply into the matter.

Old students of aeronautics will remember the " aspiration " theory that has been put forward at various times to account for soaring flight. Briefly this may be summed up by saying that the wing creates a form of vortex in the air by means of its front edge, the vortex is carried past the wing by the fluid stream, revolving so that it scrapes, as it were, against the surface of the wing, giving a forward reaction sufficient to overcome the resistance caused by the generation of the vortex in the first instance at the front edge of the wing. In old numbers of this Journal Mr. Lawrence Hargrave will be found very strongly canvassing this theory, and many since that time have dallied with the subject, without however explaining the difference between the action mooted and perpetual motion. Colonel de Villamil would be horrified if it were suggested that he was supporting the " aspiration " theory in his chapter, entitled, " Negative Resistance in Liquids," and rightly so, because although he attaches great importance to the forward reaction due to the drag of the revolving vortex, he is very careful to make clear that only a portion of the head resistance can be recuperated by the vortex action, and not the whole of it as was contended by the supporters of the " aspiration " theory. Colonel de Villamil does not refer to Mr. Hargrave's work in this chapter, nor to the experiment of Thomas Young. He does, however, refer to that of Mons. Goupil. He is apparently not aware that Mons. Goupil has admitted that there was no doubt an upward current in the case of his " aspiration " experiment. This is, however, by the way; the fact that negative resistance is possible cannot at this time of day be controverted, the phenomenon merely wants further investigation. Further investigation is, in fact, what is very badly required on all these points, and this book is a stimulant in that direction. Written by an avowed iconoclast, it has the defects of its qualities, but it is well that it has been written.

NEW BOOKS AND PUBLICATIONS.

Taschenbuch der Luftflotten. F. Rasch and W. Hormel. Munchen: J. F. Lehmann. 1915. pp. 353. Illus. Price M. 4.

*__Aircraft in the Great War.__ C. Grahame-White and Harry Harper. London: T. Fisher Unwin. 1915. pp. 346. Price 7s. 6d.

*__The Zeppelin Raid in West Norfolk.__ Holcombe Ingleby. London: Edward Arnold. 1915. pp. 31. Price 3d.

PAMPHLETS.

Wireless Transmission of Energy. Elihu Thomson. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 243-260. Figs.

Recent Developments in Electro-Magnetism. Eugene Bloch. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 223-241. Figs.

Modern Ideas on the End of the World. Gustav Jaumann. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 213-221. Figs.

The Earth and Sun as Magnets. Dr. George E. Hale. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 145-158. Plates.

The Earth's Magnetism. L. A. Bauer. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 195-212. Plates.

Problems in Smoke, Fume and Dust Abatement. F. G. Cottrell. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 653-685. Plates.

Flameless Combustion. Carleton Ellis. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 639-652. Plates.

Oil Films on Water and on Mercury. Henri Devaux. Washington: Government Printing Office. 1914. Smithsonian Report, 1913. pp. 261-273. Plates.

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* See Review in present issue.